

# Pulsed Nonlinear Surface Acoustic Waves in Crystals

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**Abstract:** Predictions from a recent theory for the propagation of nonlinear surface waves in anisotropic solids are compared with measurements of laser-generated surface-wave pulses in silicon. These are the first reported comparisons of theory and experiment for the nonlinear evolution of surface waves in a crystal.

## THEORY AND EXPERIMENT

A theoretical model was developed recently [1] that describes the propagation of plane nonlinear surface waves in anisotropic media. The spectral equations for the  $j$ th vector component ( $j = x, y, z$ ) of the particle velocity in the surface wave are

$$v_j(x, z, t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx - \omega t)}, \quad \frac{dv_n}{dx} + \alpha_n v_n = -n^2 \sum_{l+m=n} \frac{lm}{|lm|} R_{lm} v_l v_m,$$

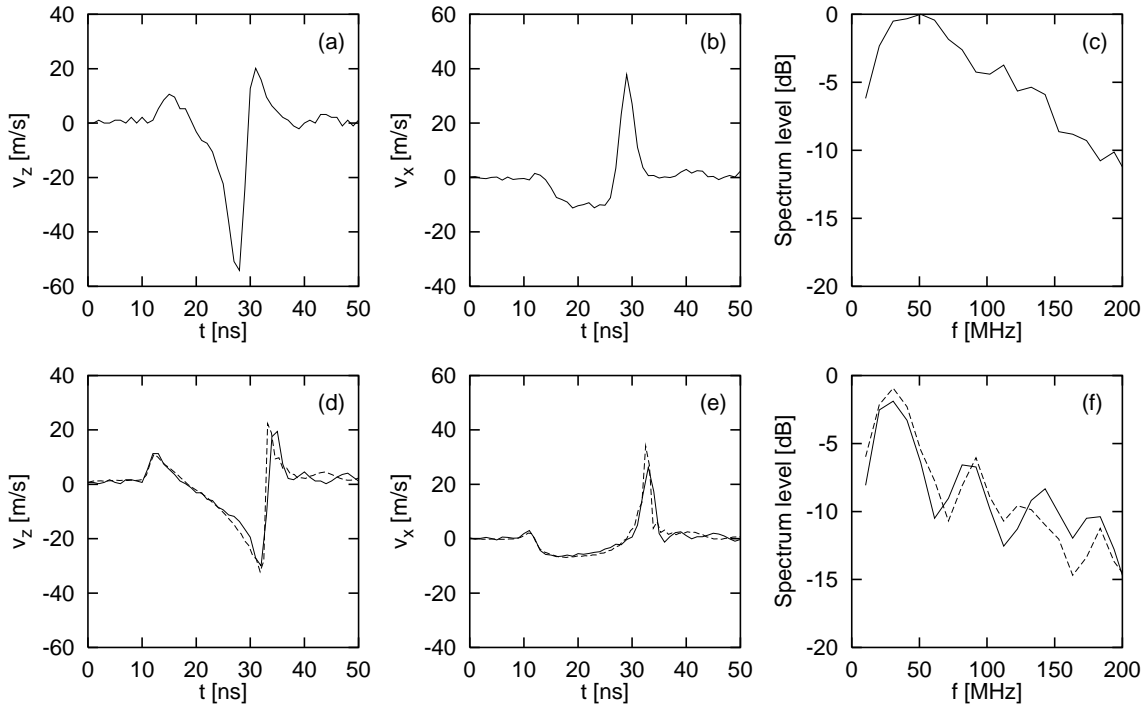
where  $x$  is the direction of propagation,  $z$  the coordinate normal to the traction-free surface of the solid,  $\omega$  the fundamental angular frequency and  $k$  the corresponding wavenumber in the expansion;  $u_{nj}(z)$  are eigenfunctions of the linear problem, and  $R_{lm}$  is a nonlinearity matrix that is known explicitly in terms of the second- and third-order elastic constants of the material. The coupled equations for  $v_n(x)$  are solved numerically, with the *ad hoc* absorption coefficients  $\alpha_n = n^2 \alpha_1$  introduced for numerical stability when shocks are formed. Numerical simulations were reported for propagation in several real crystals, and for different surface cuts and propagation directions [2]. This previous work [1,2] focused on radiation from monofrequency sources. Our purpose here is to report comparison of the theory with measurements of pulsed nonlinear surface waves in crystalline silicon. The present work thus extends an earlier comparison of theory and experiment [3] for nonlinear Rayleigh waves in an isotropic solid.

Generation and detection of the surface waves were accomplished with Nd:YAG lasers [3–5]. Laser radiation of wavelength 1064 nm, pulse duration 7 ns, and energy up to 50 mJ generated the surface waves. The radiation was focused with a cylindrical lens into a thin strip 6 mm by 50  $\mu$ m on the surface of crystalline silicon cut along its (111) plane. To enhance the conversion of optical to elastic energy, a liquid layer having a large optical absorption coefficient was deposited on the surface of the silicon in the excitation region. The surface waves were detected by the deflection of a probe laser beam (diode pumped Nd:YAG, wavelength 532 nm, power 40 mW) that irradiated spots of approximately 4  $\mu$ m in diameter on the surface of the silicon at distances 5 mm and 21 mm from the excitation region. The reflected probe signals were detected by two photodiodes, the output from which is proportional to the vertical velocity component  $v_z$  at the surface. Surface wave pulses in these experiments had durations of 20–40 ns and peak strains of order  $10^{-2}$ .

## RESULTS

Figures 1(a)–(c) show the measured waveforms and peak-normalized spectrum at distance  $x = 5$  mm from the excitation region. Propagation of the surface wave was in the  $\langle 112 \rangle$  direction of the (111) plane, and because of symmetry  $v_y = 0$ . Linear theory was used to compute the horizontal velocity  $v_x$  [1(b)] from the measured vertical velocity  $v_z$  [1(a)]. The measurement at  $x = 5$  mm was used as the starting condition for the computations, and the resulting predictions for the waveforms and spectra at  $x = 21$  mm are shown as dashed lines in Figs. 1(d)–(f). The solid lines again correspond to the measurements. No curve fitting was employed—all material constants used in the calculations were taken directly from measurements reported

in the literature [6]. The nonlinear waveform distortion is predicted accurately by the theory, including the increase in pulse duration (note that the spectral peak shifts from 50 MHz down to about 30 MHz). There are notable differences with the Rayleigh wave measurements reported earlier [3–5]. In the present experiment, the vertical component  $v_z$  evolves into an N-shaped waveform and the horizontal component  $v_x$  evolves into one that is U-shaped, whereas the reverse was observed in isotropic solids.



**FIGURE 1.** Comparison of experiment (solid lines) and theory (dashed lines) for surface waves propagating in the (111) plane of crystalline silicon, from  $x = 5$  mm (upper row) to  $x = 21$  mm (lower row) in the  $\langle 112 \rangle$  direction.

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