PULSED NONLINEAR SURFACE ACOUSTIC WAVES IN CRYSTALS

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OUTLINE

- Anisotropy in Crystalline Silicon
- Theory
- Experiment
- Diffraction Effects
- Simulations with Sinusoids
- Comparison of Experiment and Theory
- Conclusion & Future Work

ANISOTROPY IN CRYSTALLINE SILICON

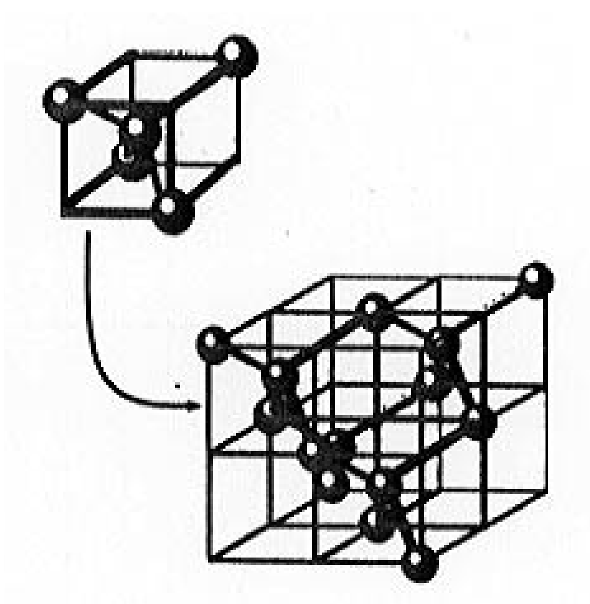
Stress-strain relation for cubic crystal:

$$\begin{aligned}\sigma_{ij} &= c_{ijkl}e_{kl} + d_{ijklmn}e_{kl}e_{mn} \\ c_{ijkl} &\rightarrow \text{3 Second Order Elastic (SOE) constants} \\ d_{ijklmn} &\rightarrow \text{6 Third Order Elastic (TOE) constants}\end{aligned}$$

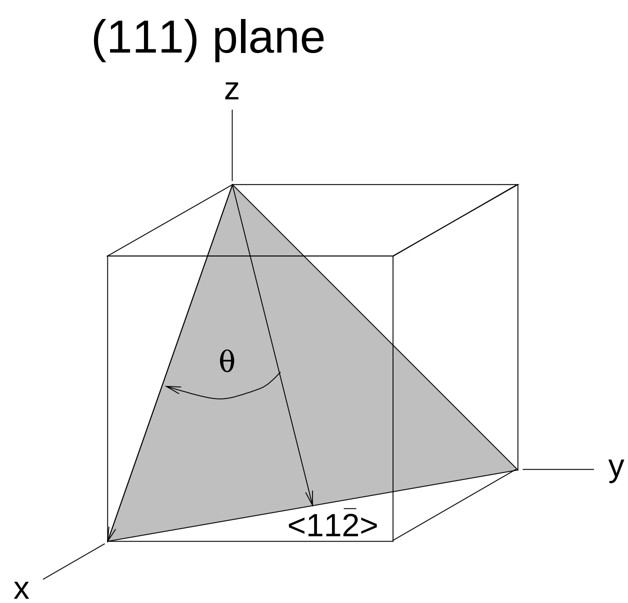
Data for Si elastic constants:

- McSkimin, H. J. and Andreatch, Jr., P., *J. Appl. Phys.* **35**, 3312–3319 (1964).

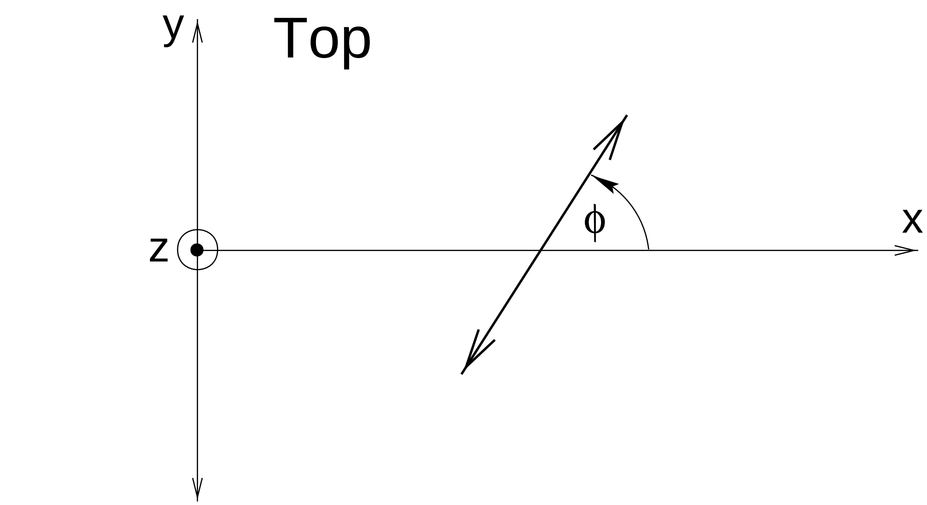
Diamond Cubic Structure:

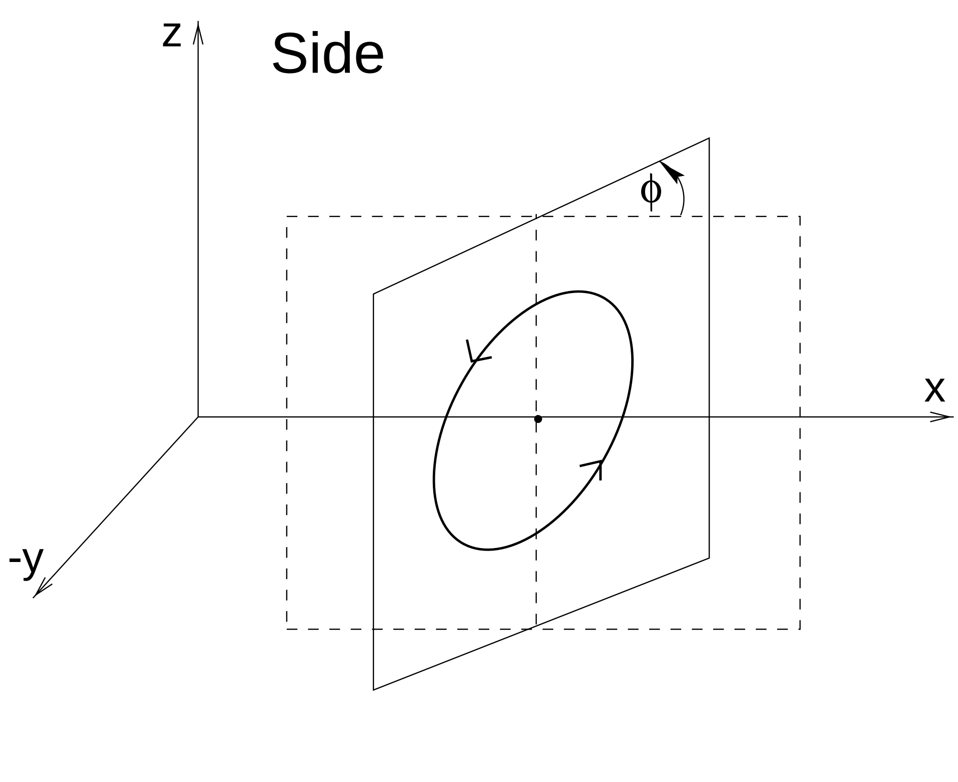


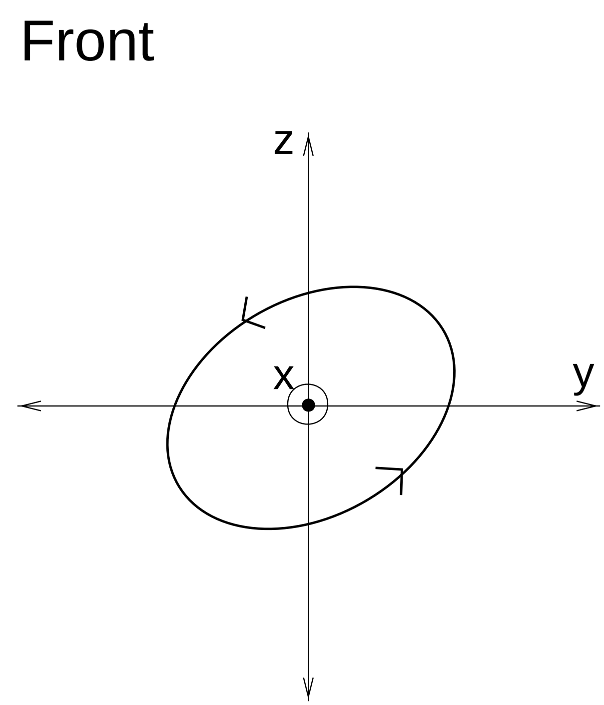
Crystal Cut in Experiment:



Schematic Diagram of a Surface Acoustic Wave in an Anisotropic Medium







THEORY

Approach: Hamiltonian mechanics formalism
(Hamilton, Il'inskii, Zabolotskaya, 1996)

Velocity waveforms in solid:

$$v_j(x,z,t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)}$$

Coupled spectral evolution equations:

$$\frac{dv_n}{dx} + \alpha_n v_n = -n^2 \sum_{l+m=n} \frac{lm}{|lm|} R_{lm} v_l v_m$$

Solve equations numerically:

- Input data

  Material constants (density, SOE, TOE)

  Waveform spectrum ($x = 5$ mm)

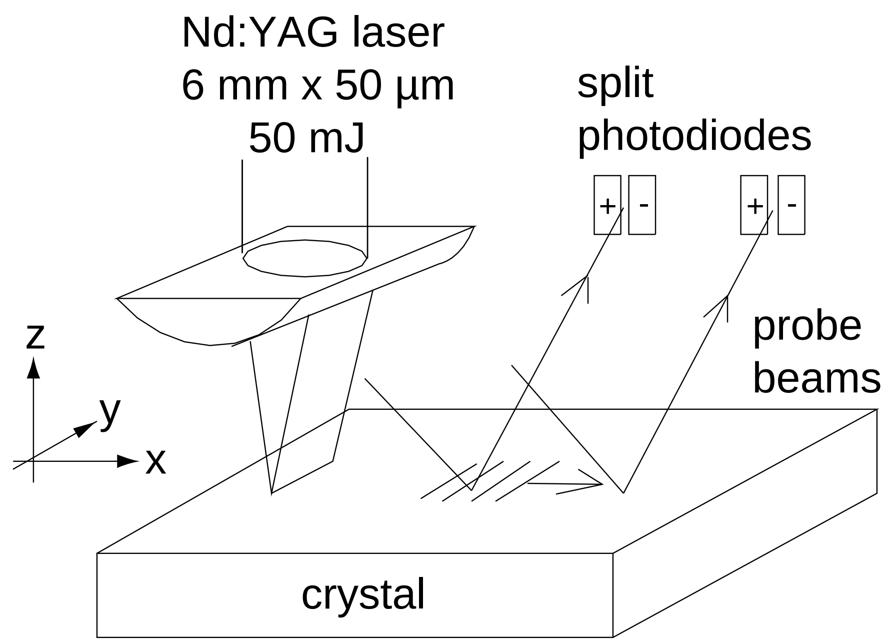
- Apply 4th order Runge-Kutta routine with:

Number of harmonics:	400
Pulse repetition frequency:	10 MHz
Maximum bandwidth:	4000 MHz

Weak absorption was added for numerical stability.

EXPERIMENT

Approach: Laser-excited thermoelastic SAW generation (Lomonosov and Hess, 1996)



- Pulse detection:

  Probe beam deflection proportional to vertical vel.

  Photodiode bandwidth: 500 MHz

- Beam locations:

  Laser excitation: $x = 0$ mm

  1st probe beam: $x = 5$ mm

  2nd probe beam: $x = 21$ mm

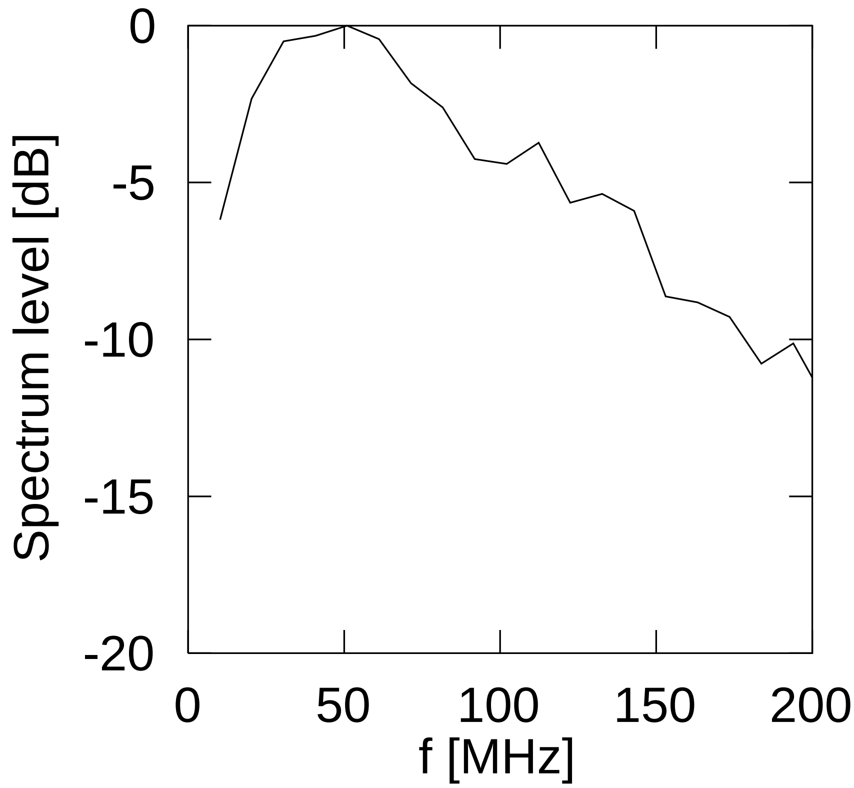
- Resulting SAW pulses:

  Duration: 20 to 40 ns

  Peak strain: 0.005 to 0.010 (near fracture)

DIFFRACTION EFFECTS

Measured frequency spectrum at $x = 5$ mm:

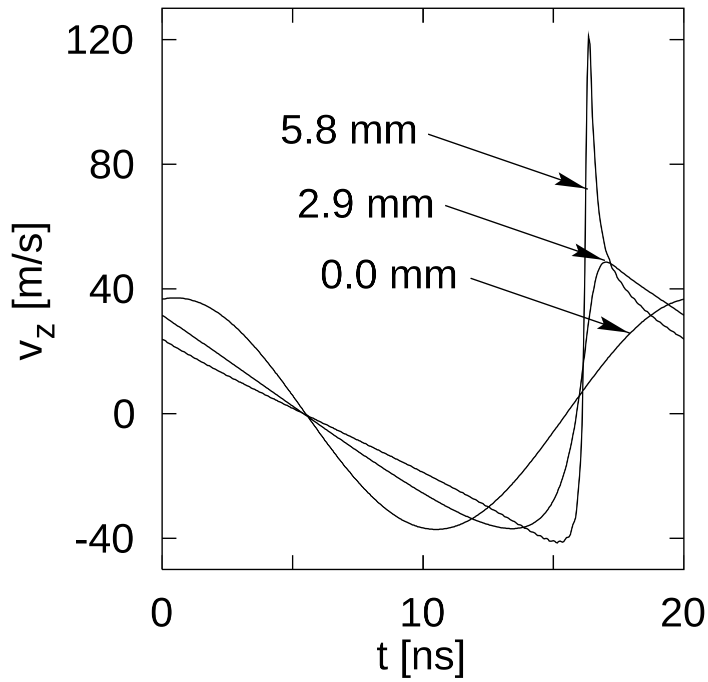


Characteristic frequency: 50 MHz

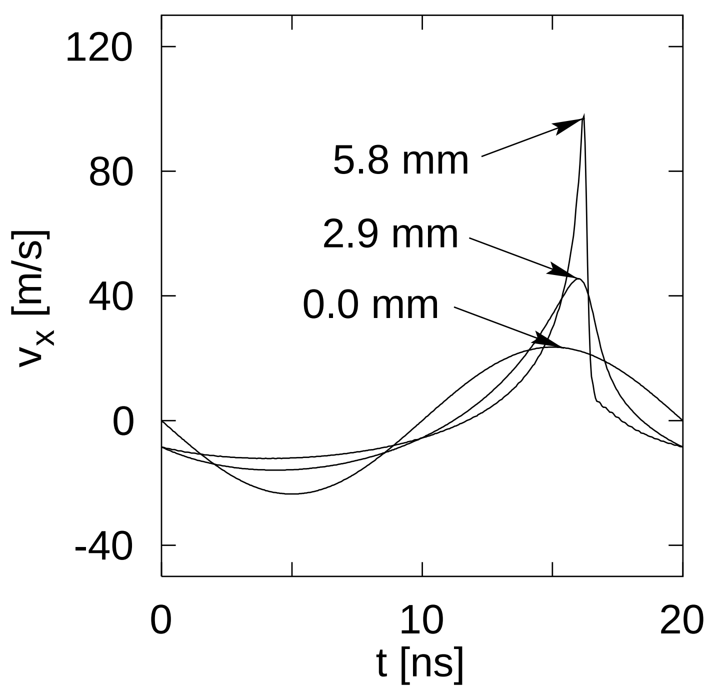
- Analysis:
  Characteristic beam radius: $a = 3$ mm
  Diffraction length for SAW beam: $\frac{1}{2}ka^2 = 300$ mm
  Furthest measurement distance: $x = 21$ mm
- Conclusion:
  Diffraction effects are not important.

SIMULATIONS WITH SINUSOIDS

Calculated vertical velocity waveforms ($f_0 = 50$ MHz):

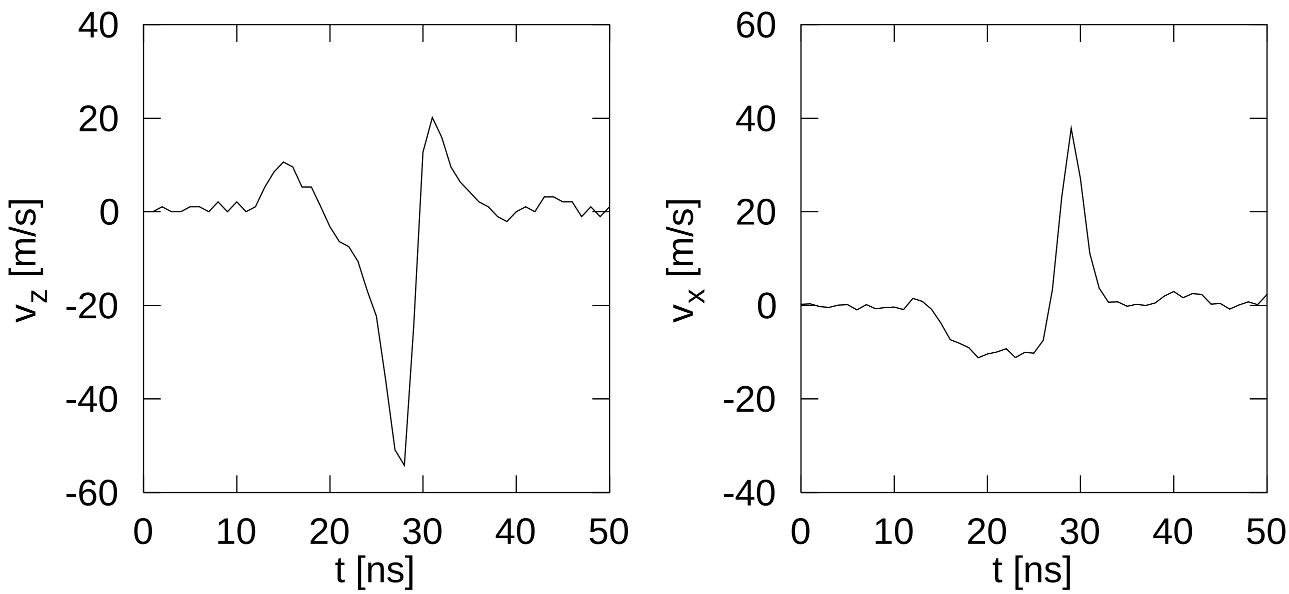


Calculated longitudinal velocity waveforms:

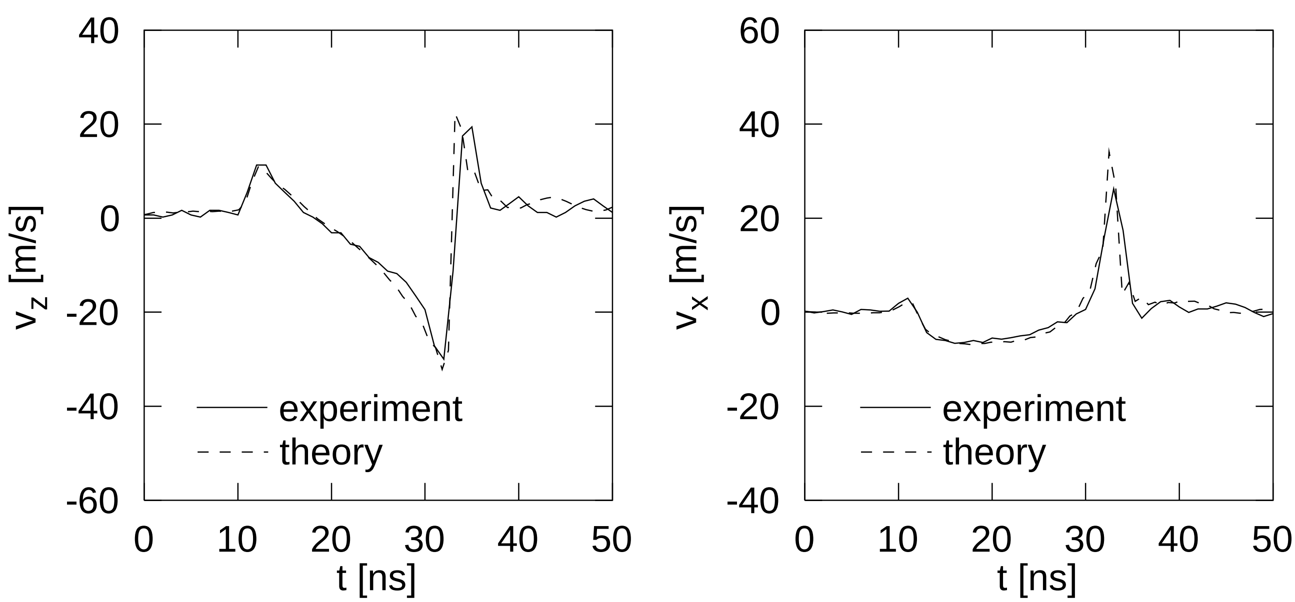


EVOLUTION OF WAVEFORMS

Velocity waveforms at $x = 5$ mm:



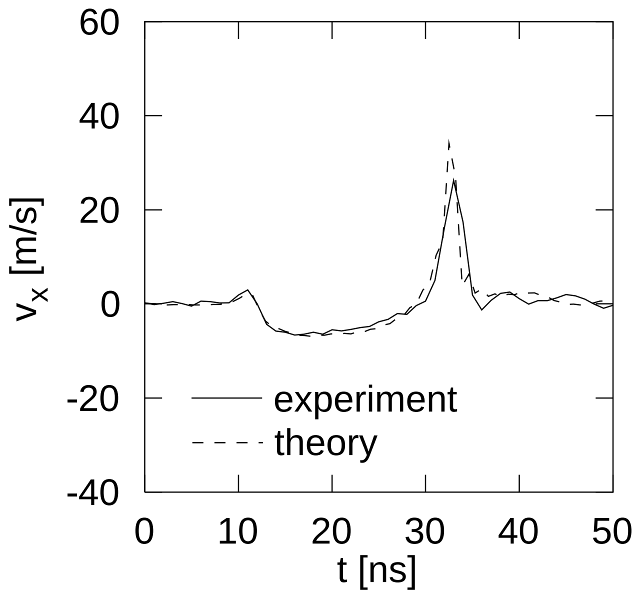
Velocity waveforms at $x = 21$ mm:

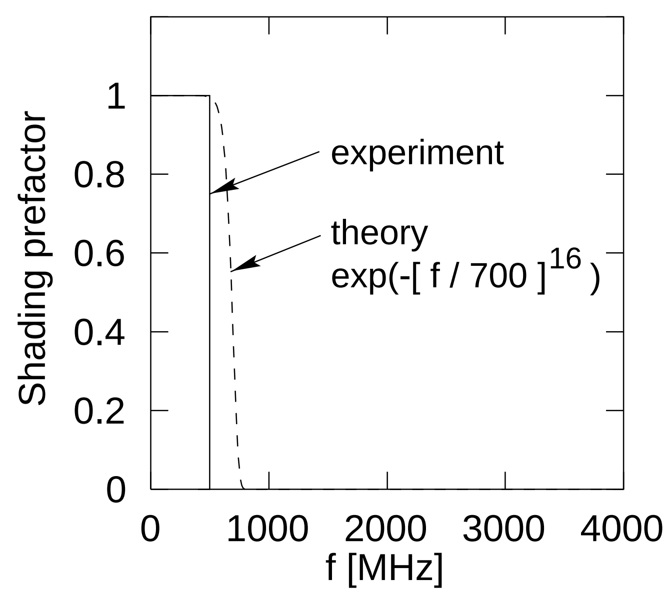


EFFECT OF RECONSTRUCTION BANDWIDTH

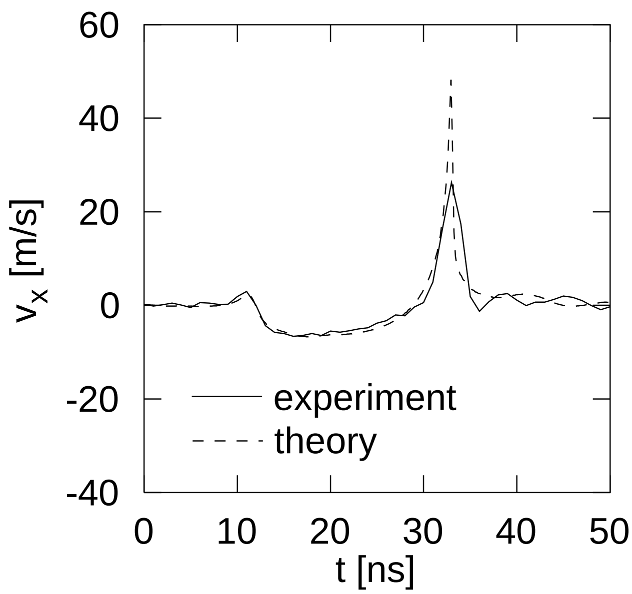
Consider the longitudinal velocity waveforms at $x = 21$ mm.

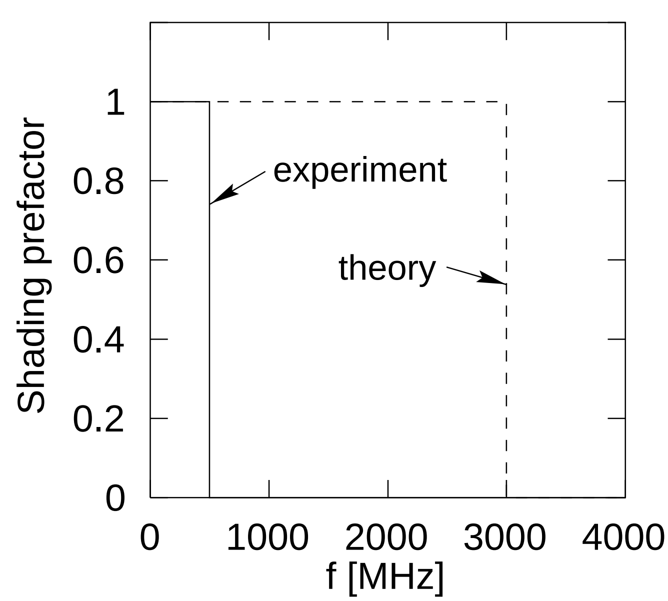
Theoretical waveforms with shaded bandwidth of 700 MHz:





Theoretical waveforms with bandwidth of 3000 MHz:





CONCLUSION & FUTURE WORK

Results:

- First reported comparison of experiment and theory for nonlinear SAW in a crystal
- Theory in close quantitative agreement with experiment
- Predictions based on fundamental material properties

Future work:

- Study relationship between nonlinearity matrix elements and waveform distortion [Norfolk ASA meeting]
- Study variation of waveform evolution as function of direction and cut
- Investigate other anisotropic materials
- Investigate piezoelectric effects

NONLINEARITY MATRIX & LINEAR THEORY

The nonlinearity matrix is given by

$$R_{n_1 n_2} = -\sum_{s_1,s_2,s_3=1}^{3} \frac{d'_{iklmpq}\beta_i^{(s_1)}\beta_l^{(s_2)}\beta_p^{(s_3)*}l_k^{(s_1)}l_m^{(s_2)}l_q^{(s_3)*}}{2[n_1 l_3^{(s_1)} + n_2 l_3^{(s_2)} + (n_1+n_2)l_3^{(s_3)*}]}$$

where $\beta_i^{(s)} = C_s \alpha_i^{(s)}$ and

$$d'_{iklmpq} = d_{iklmpq} + c_{ikmq}\delta_{lp} + c_{lmkq}\delta_{ip} + c_{pqkm}\delta_{il} \ .$$

To compute this expression, the linear problem must first be solved.

Start with linearized wave equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} = c_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} \ . \tag{1}$$

Next assume SAW solution of form

$$u_i = \sum_{s=1}^{3} C_s \alpha_i^{(s)} e^{ik(\mathbf{l_s}\cdot\mathbf{r}-\omega t)} \tag{2}$$

where $l_s = \{1, 0, \zeta\}$. Substitute Eq. (2) into Eq. (1) to yield

$$\rho c^2 \alpha_i = \tilde{c}_{ijkl} l_j l_l \alpha_k \ . \tag{3}$$

Solve Eq. (3) subject to the stress-free surface bound. cond.

$$\sigma_{i3}\,\big|_{x_3=0} = 0 \ . \tag{4}$$

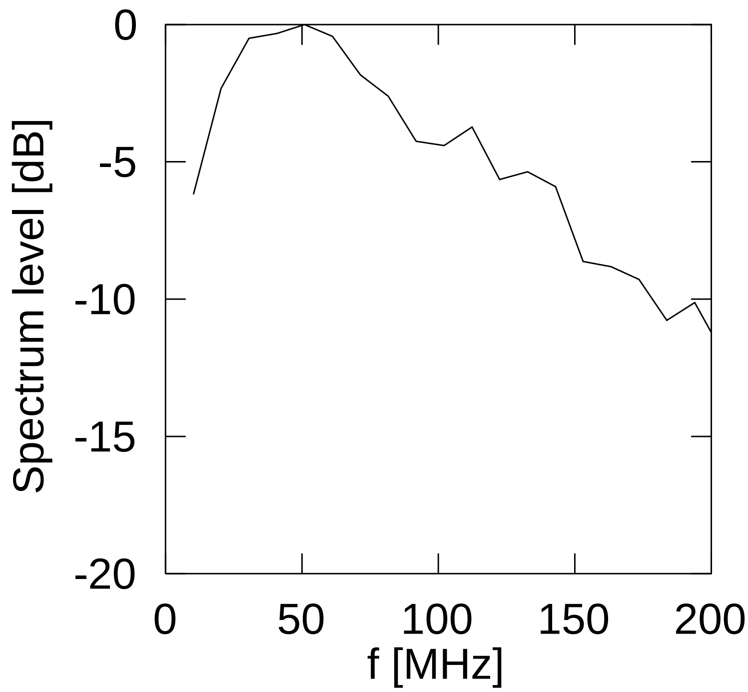
Substituting Eq. (2) into Eq. (4) yields

$$ik\tilde{c}_{i3kl}\sum_{s=1}^{3} C_s \alpha_k^{(s)}(c) l_l^{(s)} = 0 \ . \tag{5}$$

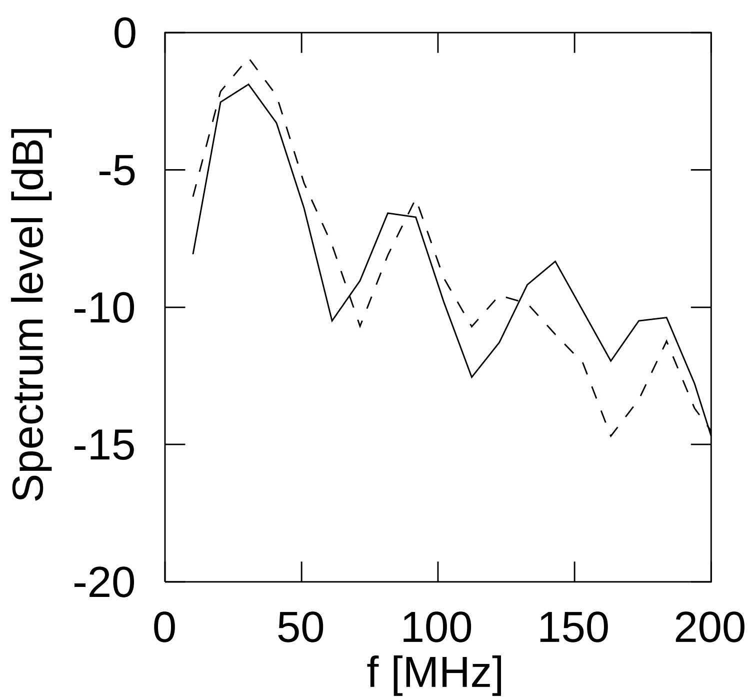
These equations can be solved numerically for $l_i^{(s)}$, $\alpha_i^{(s)}$, and C_s.

EVOLUTION OF SPECTRA

Measured spectrum at $x = 5$ mm:



Spectra at $x = 21$ mm:



NONLINEARITY PARAMETERS

Plane wave shock formation distance for sinusoid:

$$\overline{x} = \frac{1}{|\beta_x|\epsilon_x k}$$

where $\epsilon_x = v_{x0}/c$ and $v_x = v_{x0} \sin \omega t$ at $x = 0$.

- Calculated shock formation distance:
  (v_{x0}=25 m/s, $\omega/2\pi$=50 MHz)

  $$\overline{x} = 2.9 \text{ mm}$$

  Meaning:
  Because propagation distance $\Delta x = 16$ mm,
  the pulse is well into the shock formation region.

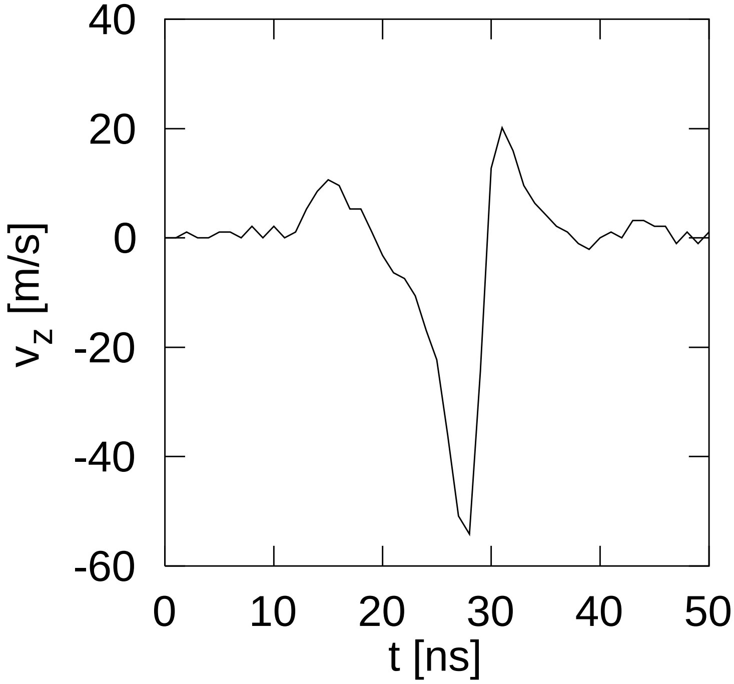
- Calculated coefficient of nonlinearity:

  $$\beta_x = -1.0$$

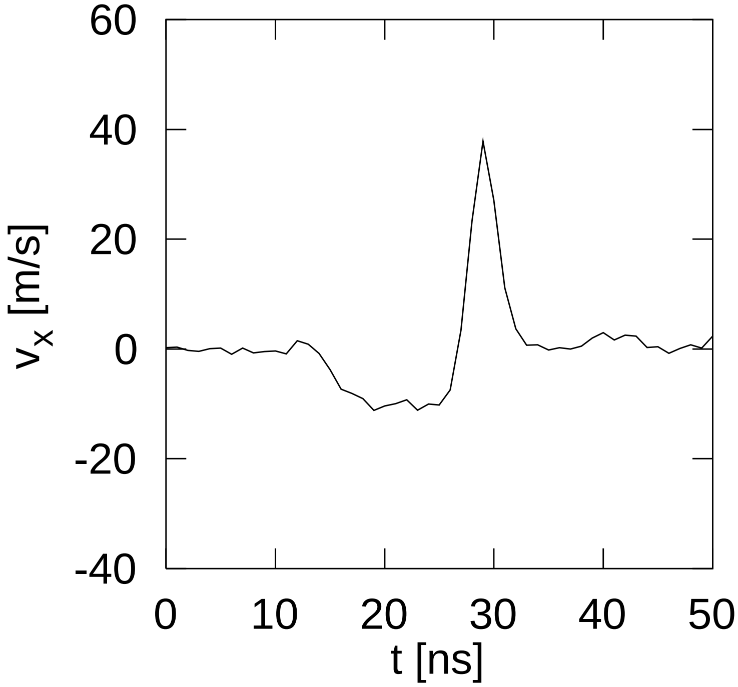
  Meaning:
  The negative sign indicates that peaks recede and troughs advance in time, opposite to the distortion of a sound wave.

WAVEFORMS AT FIRST LOCATION

Measured vertical velocity waveform at $x = 5$ mm:



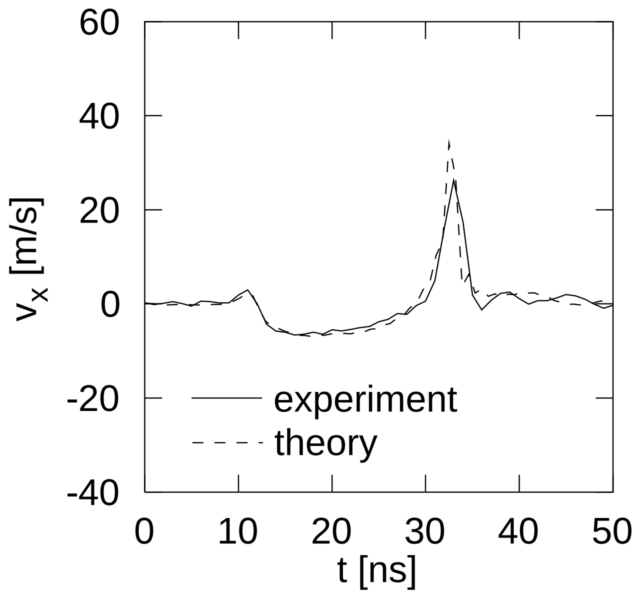
Calculated longitudinal velocity waveform from linear theory:

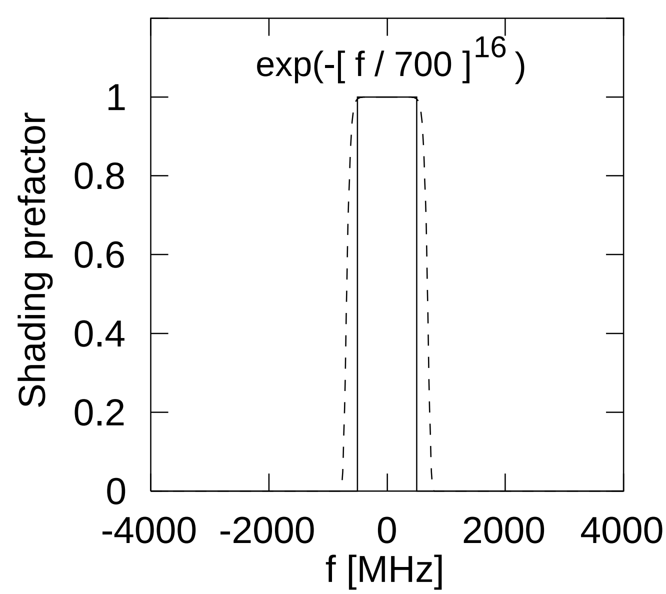


EFFECT OF RECONSTRUCTION BANDWIDTH

Consider the vertical velocity waveforms at $x = 21$ mm.

Theoretical waveforms with shaded bandwidth of 700 MHz:





Theoretical waveforms with bandwidth of 3000 MHz: