# A SIMPLE METHOD FOR COMPARING WAVEFORM DISTORTION OF NONLINEAR SURFACE WAVES IN DIFFERENT CUBIC CRYSTALS

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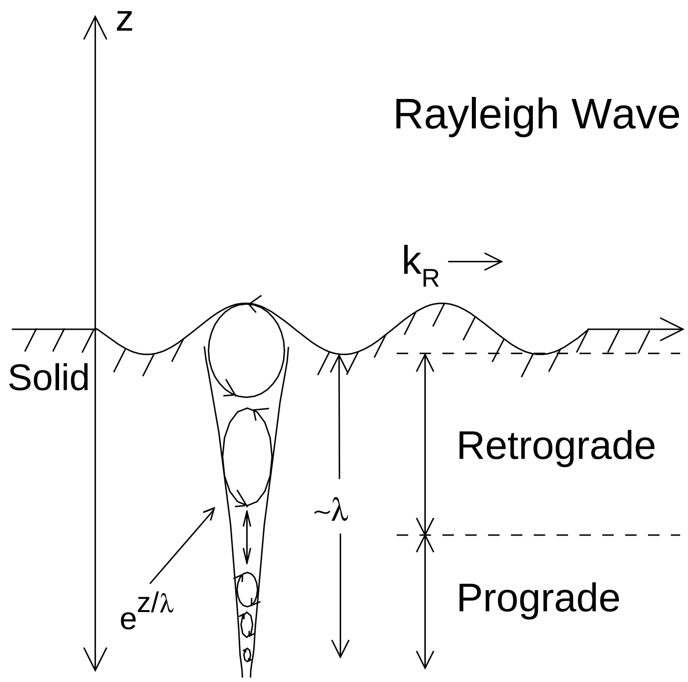
Paper 5aPA8
139th Meeting of the Acoustical Society of America
Atlanta, Georgia
30 May–03 June 2000

# OUTLINE

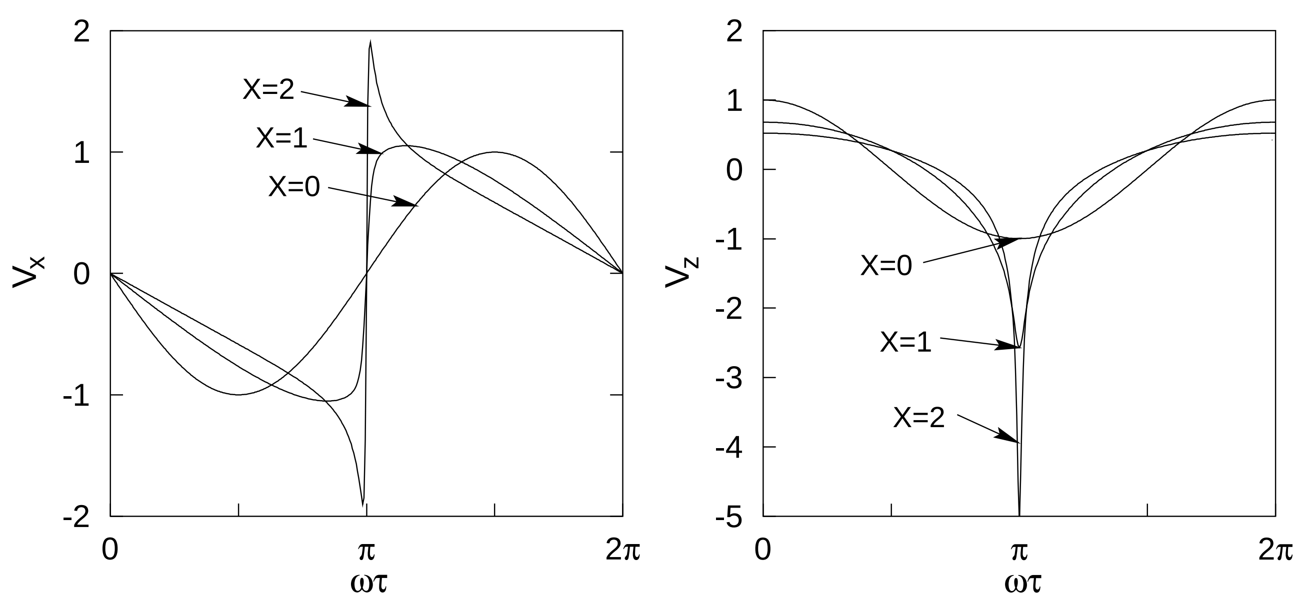
- Nonlinear Surface Waves
- Nonlinear Theory
- Comparison with Experiment
- Diversity of Nonlinear Distortion in Crystals
- Simple Approximate Method
- Comparison of Full and Approximate Methods

# NONLINEAR SURFACE WAVES

Schematic Diagram:




Waveform Distortion:
(in isotropic media or certain mirror planes of cubic crystals)



# NONLINEAR THEORY

Approach: Hamiltonian mechanics formalism
(Hamilton, Il'inskii, Zabolotskaya, 1996)

Velocity waveforms in solid:

$$v_j(x,z,t) \;=\; \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)}$$

$$u_{nj}(z) \;=\; \sum_{s=1}^{3} q_j^{(s)} e^{inkl_3^{(s)} z}$$

Coupled spectral evolution equations:

$$\frac{dv_n}{dx} + \alpha_n v_n = \frac{n^2\omega}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} S_{lm} v_l v_m$$

$v_n \;\rightarrow\;$ $n$th harmonic amplitude

$S_{lm} \;\rightarrow\;$ nonlinearity matrix elements

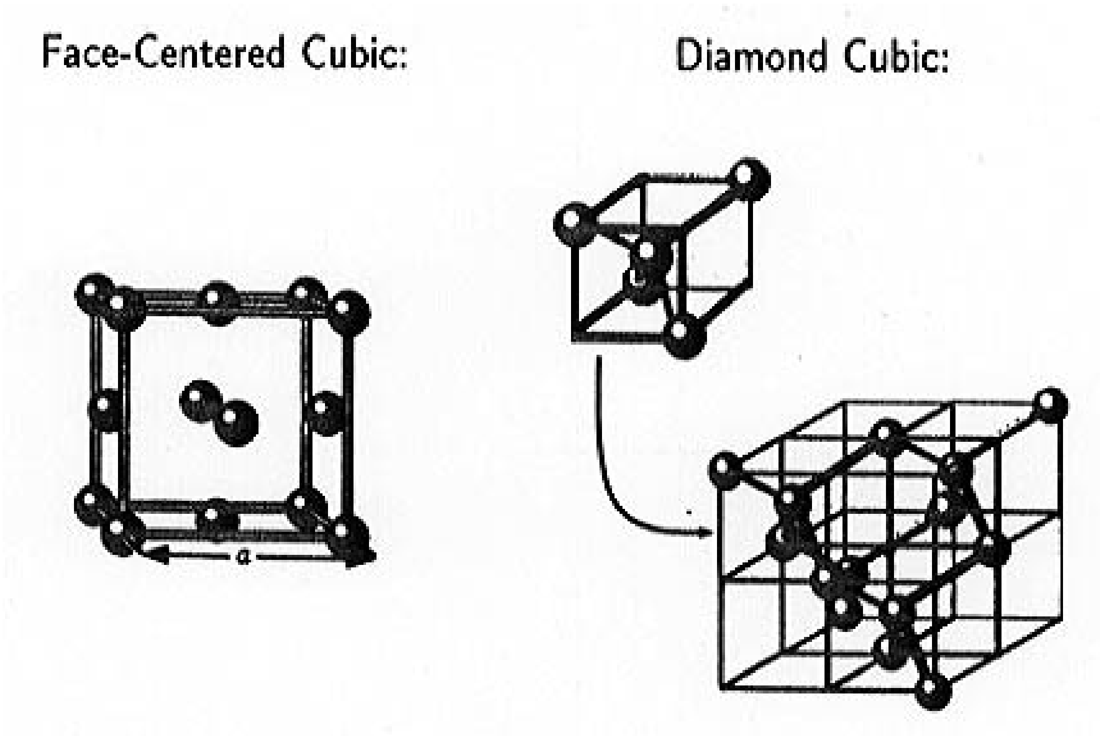
$\alpha_n \;\rightarrow\;$ weak attenuation

Observation:

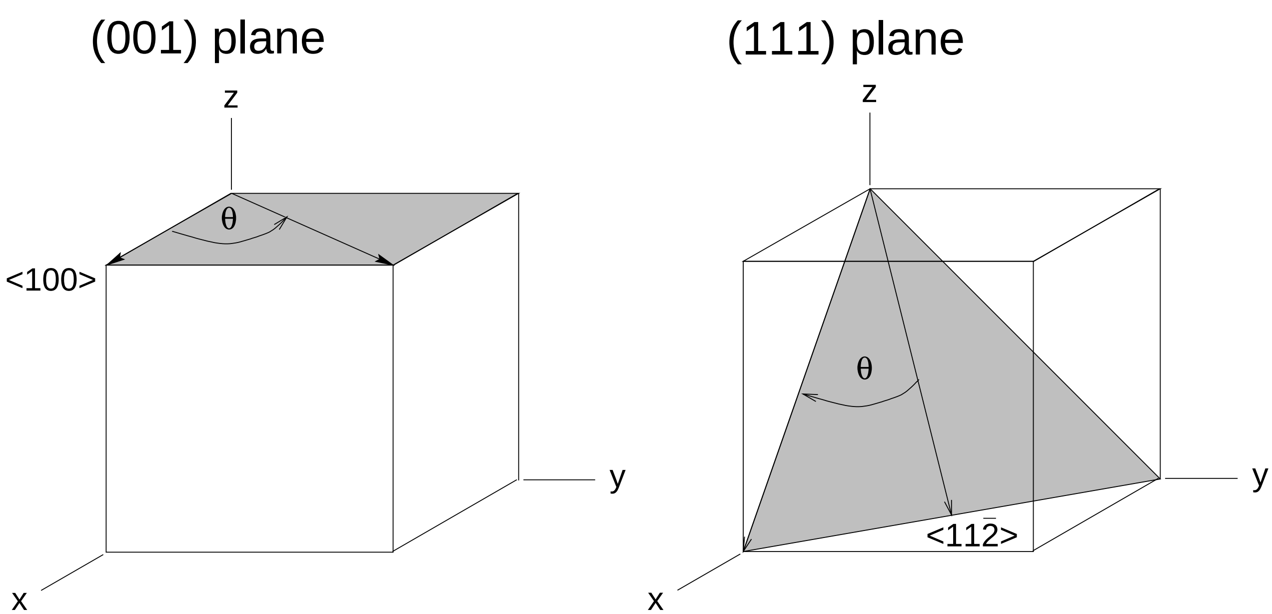
Higher symmetry cases $\Rightarrow$ $S_{lm}$ real valued

Lower symmetry cases $\Rightarrow$ $S_{lm}$ complex valued

# CRYSTALLINE GEOMETRY




Typical Planes for Crystal Cuts:

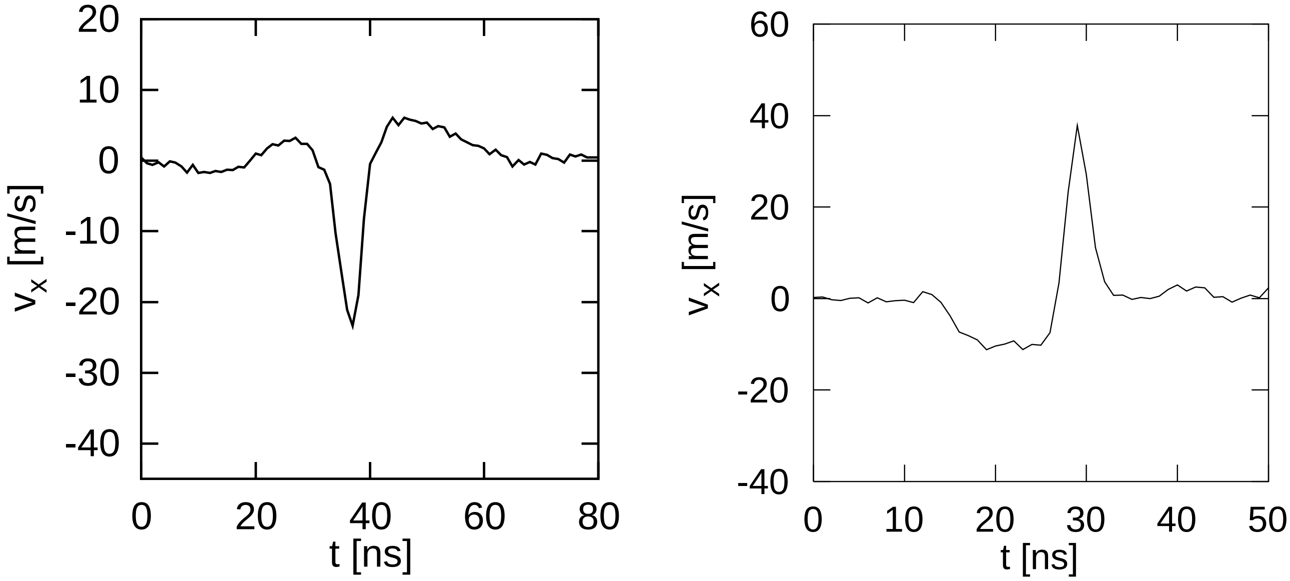


# COMPARISON WITH EXPERIMENT

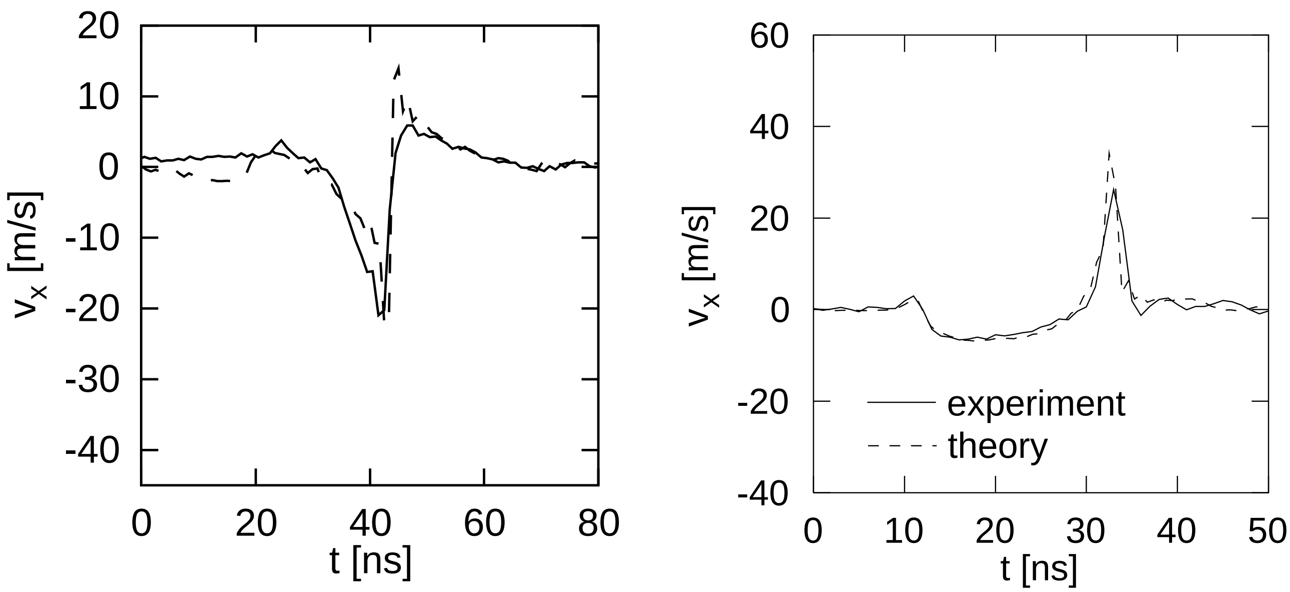
[data from A. Lomonosov and P. Hess (1998, 1999)]

Si, (001) plane, $26^\circ$ from $\langle 100 \rangle$ Si, (111) plane, $0^\circ$ from $\langle 11\bar{2} \rangle$

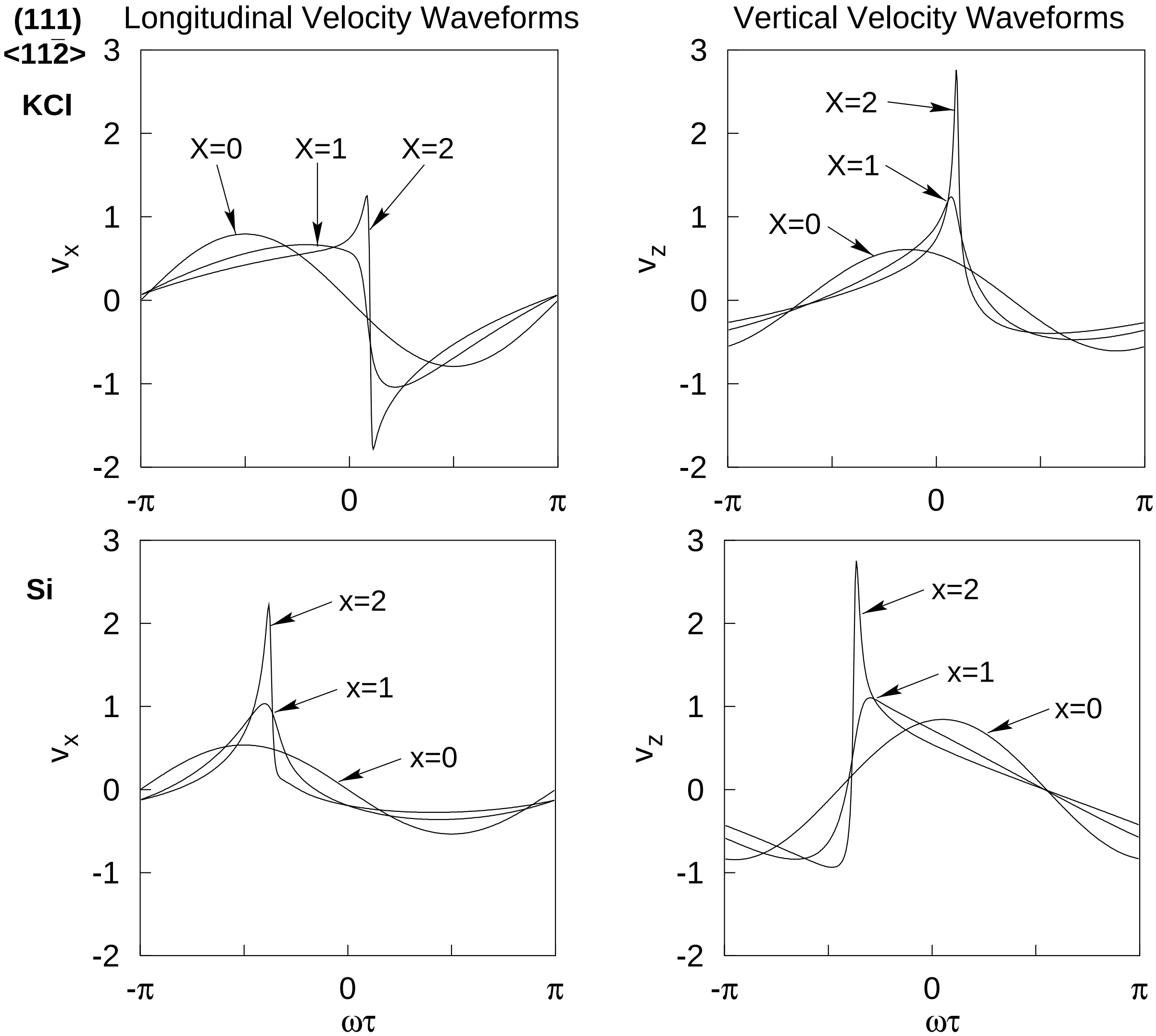
Velocity waveforms at close location:

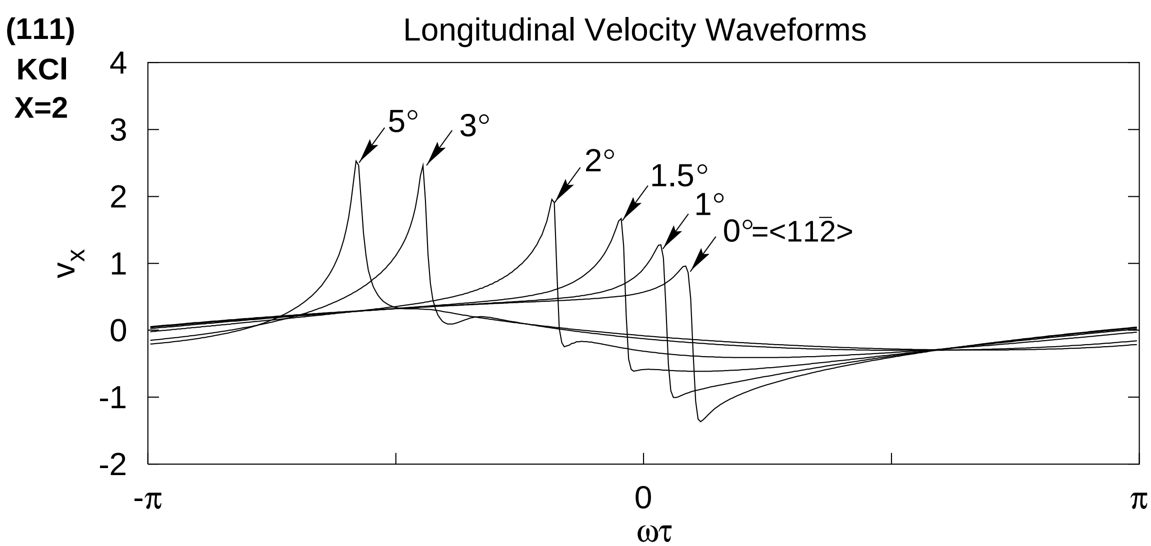



Velocity waveforms at remote location:



# WAVEFORM DISTORTION IN (111) PLANE



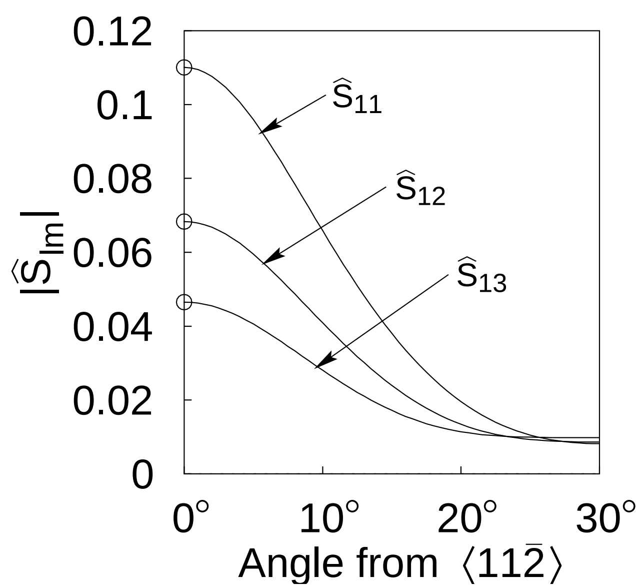



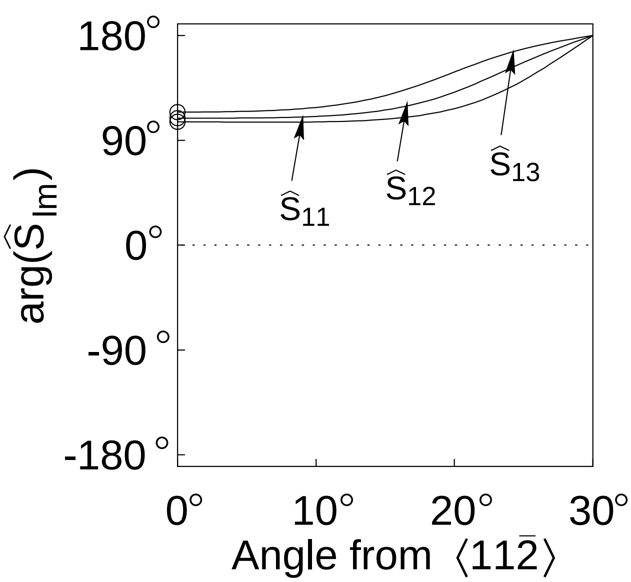
# NONLINEARITY MATRIX

Nonlinearity matrix is:

- Only a function of density, 2nd and 3rd order elastic constants, and the eigenvalues and eigenvectors of the linear problem,
- Simpler to compute than a full integration of model eqs.

Si (111) Magnitude: Phase:




Query:
How is the phase of the matrix elements related to the the shape of waveform distortion?

# PHASE OF MATRIX ELEMENTS

Define the transformation

$$S_{lm}^{\theta} = S_{lm} e^{i(\operatorname{sgn} n)\theta} , \qquad n = l + m . \tag{1}$$

The evolution equations with the transformed matrix are

$$\frac{dv_n^{\theta}}{dx} = \frac{n^2 \omega_0}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} S_{lm}^{\theta} v_l^{\theta} v_m^{\theta} .$$

Defining the relation

$$v_n^{\theta} = v_n e^{-i(\operatorname{sgn} n)\theta} \tag{2}$$

implies that the evolution equations can be rewritten as

$$\frac{dv_n}{dx} = \frac{n^2 \omega_0}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} S_{lm} v_l v_m .$$

Result:
Eqs. (1) and (2) show how the phase of the matrix elements are related to the phase of the spectral components (and therefore the waveform shape).

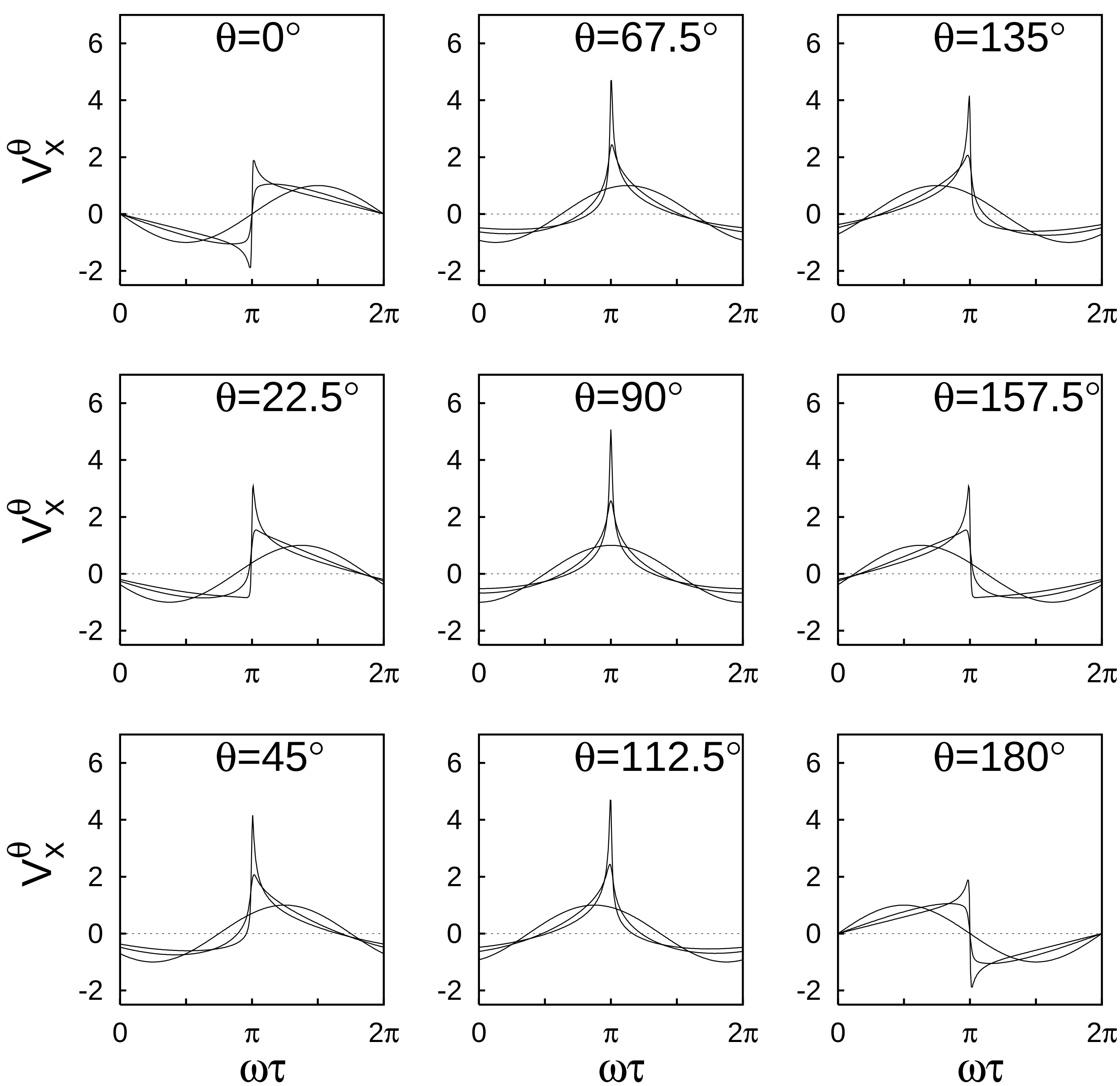
# PHASE TRANSFORMED WAVEFORMS

Consider Rayleigh waves in steel

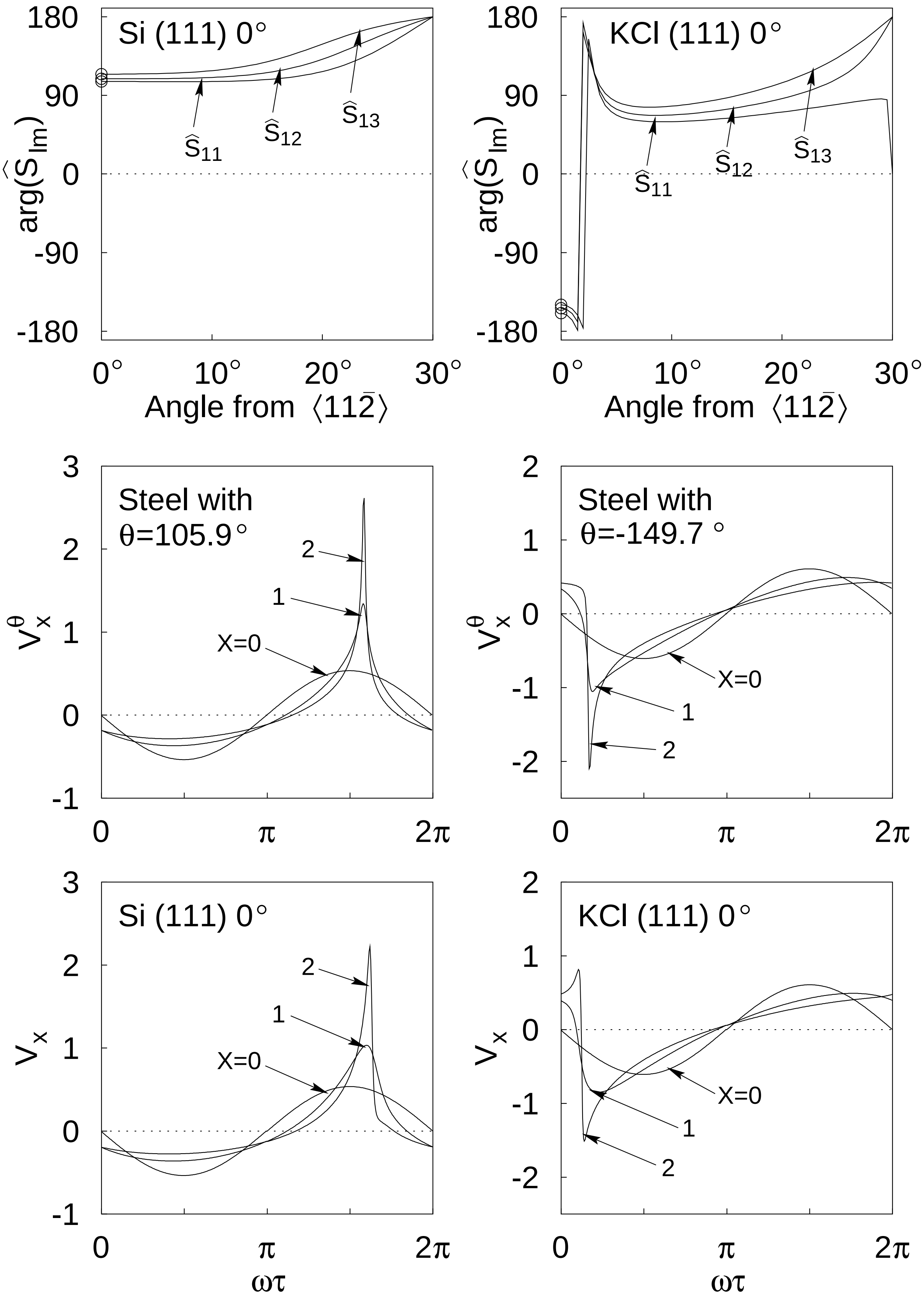
$$S_{lm} \quad \longrightarrow \quad \text{positive, real valued matrix elements}$$
$$v_n \quad \longrightarrow \quad \text{corresponding spectral components}$$

under the transformation $v_n^\theta = v_n e^{-i(\operatorname{sgn} n)\theta}$.

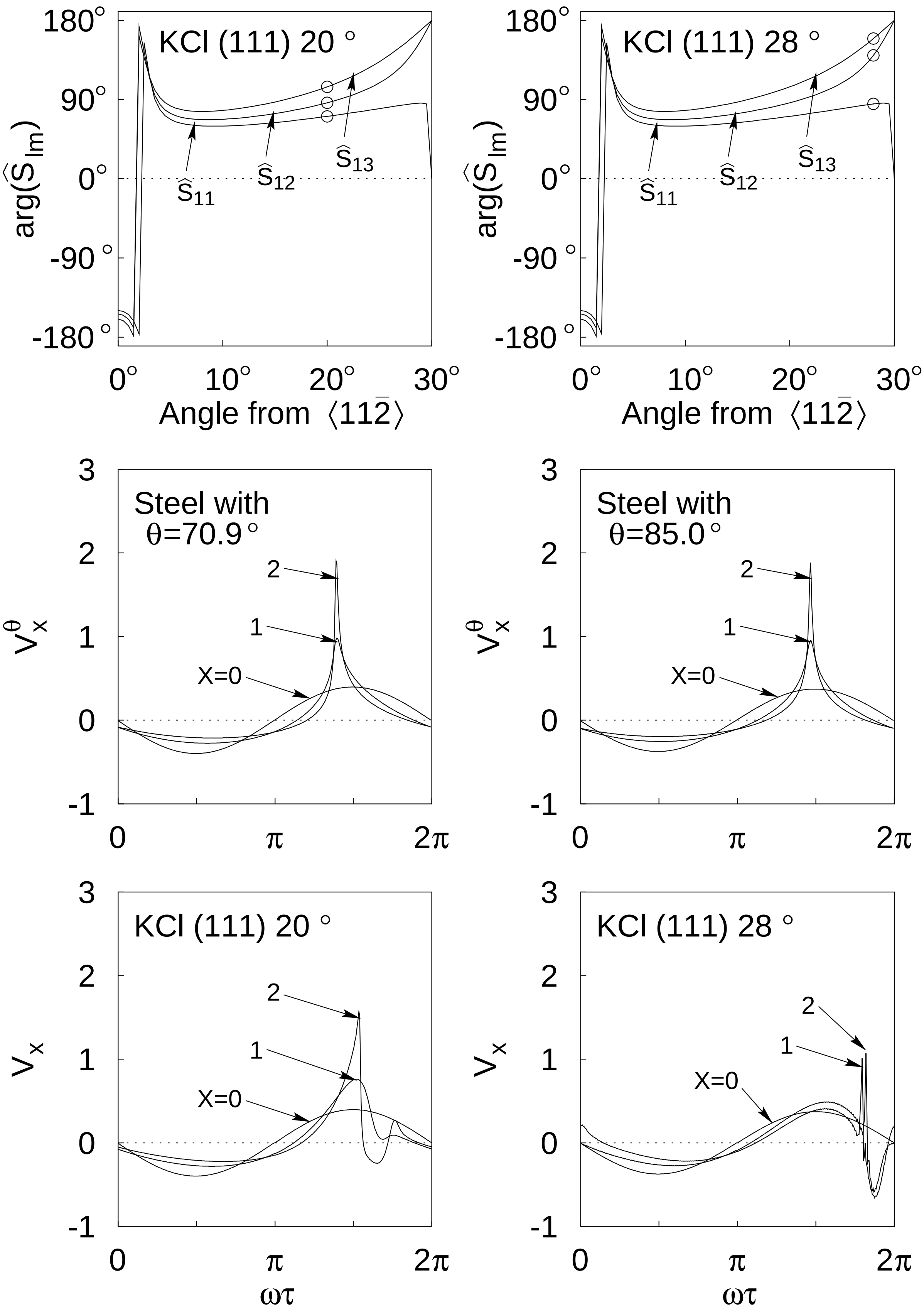
Resulting longitudinal velocity waveforms:



# ELEMENTS WITH SIMILAR PHASE



# ELEMENTS WITH DISSIMILAR PHASE



# CONCLUSION

Results:

- The phase of the nonlinearity matrix elements was shown to be key to describing waveform distortion in cubic crystals.

- An approximate method for characterizing waveform evolution using this phase was developed.

- For similarly phased matrix elements, the dominant matrix element provided a good approximation of waveform distortion.

- For dissimilarly phased matrix elements, the full solution of the model equations must be performed.