PERIODICITY OF LINEAR AND NONLINEAR SURFACE ACOUSTIC WAVE PARAMETERS IN THE (111) PLANE OF CUBIC CRYSTALS

A. Lomonosov and P. Hess
Institute of Physical Chemistry
University of Heidelberg, Heidelberg, Germany

R. E. Kumon
National Institute of Standards and Technology
Boulder, Colorado

M. F. Hamilton
Department of Mechanical Engineering
The University of Texas at Austin, Austin, Texas

Paper 3aPAb5
142th Meeting of the Acoustical Society of America
Fort Lauderdale, Florida
03-07 December 2001

J. Acoust. Soc. Am. **110**, 2702(A) (2001)

OUTLINE

- Background
  - Surface acoustic waves (SAWs) in crystals
  - Periodicity of linear parameters

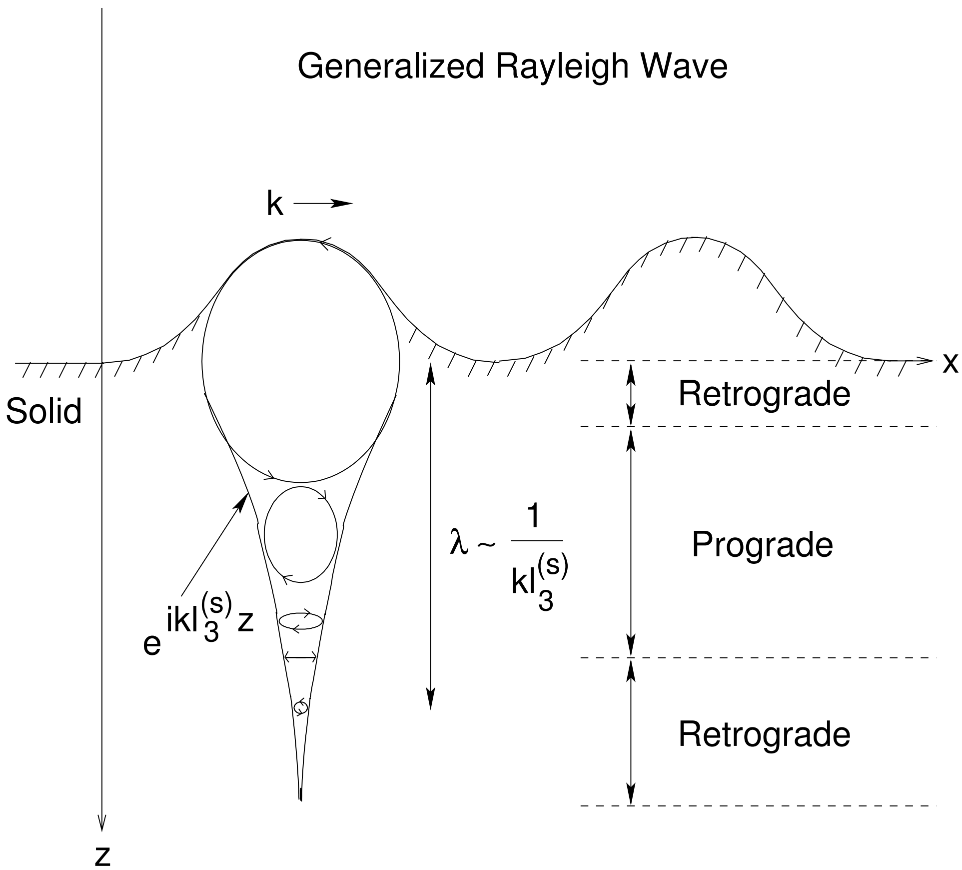
- Experiment: Nonlinear SAWs
  - Periodicity of nonlinear parameters (different!)

- Theory: Nonlinear SAWs
  - Model equations and nonlinearity parameters
  - Comparison with experiment

- Sources of periodicity differences

SURFACE WAVES IN CRYSTALS

Schematic Diagram:



Velocity components:

$$v_j(x,z,t) = \sum_{s=1}^{3} \beta_j^{(s)} \exp[ikl_3^{(s)}z] \exp[ik(x-ct)]$$

$k \;\longrightarrow$ wave number

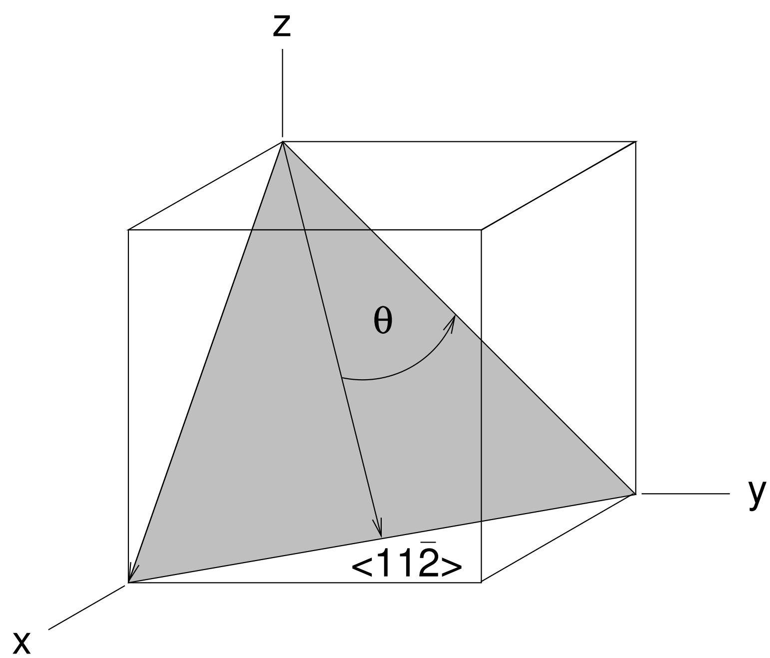
$c \;\longrightarrow$ surface wave speed

$l_j^{(s)} \;\longrightarrow$ depth penetration coefficients

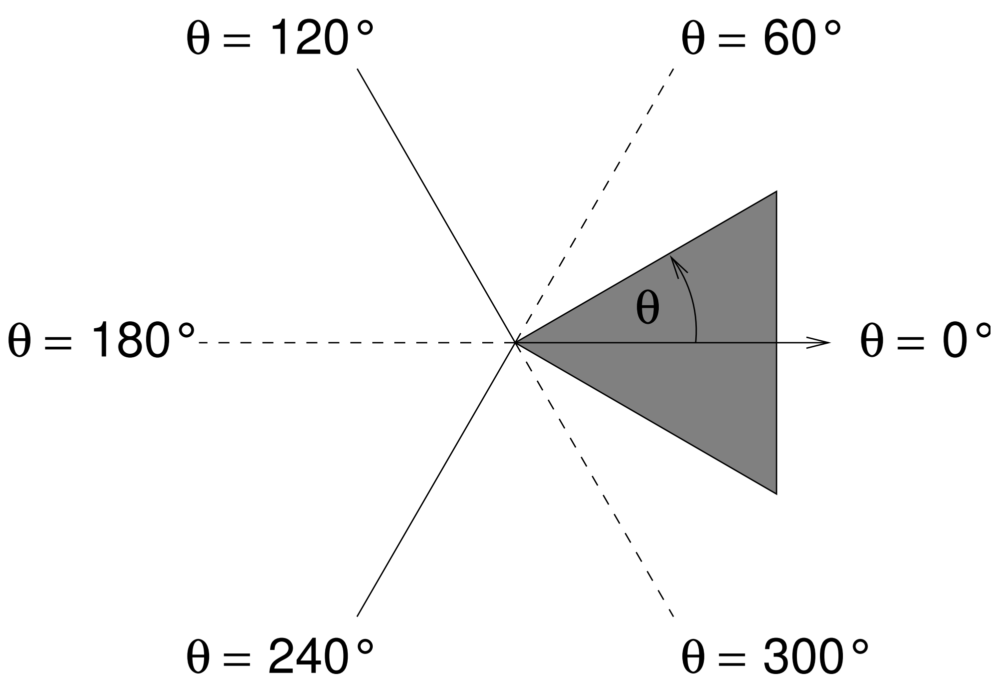
$\beta_j^{(s)} \;\longrightarrow$ amplitude coefficients

GEOMETRY AND SYMMETRY OF (111) PLANE

Surface cut and propagation directions:



Full range of directions in plane:



PERIODICITY OF LINEAR PARAMETERS

[Farnell, *Physical Acoustics*, 1970]

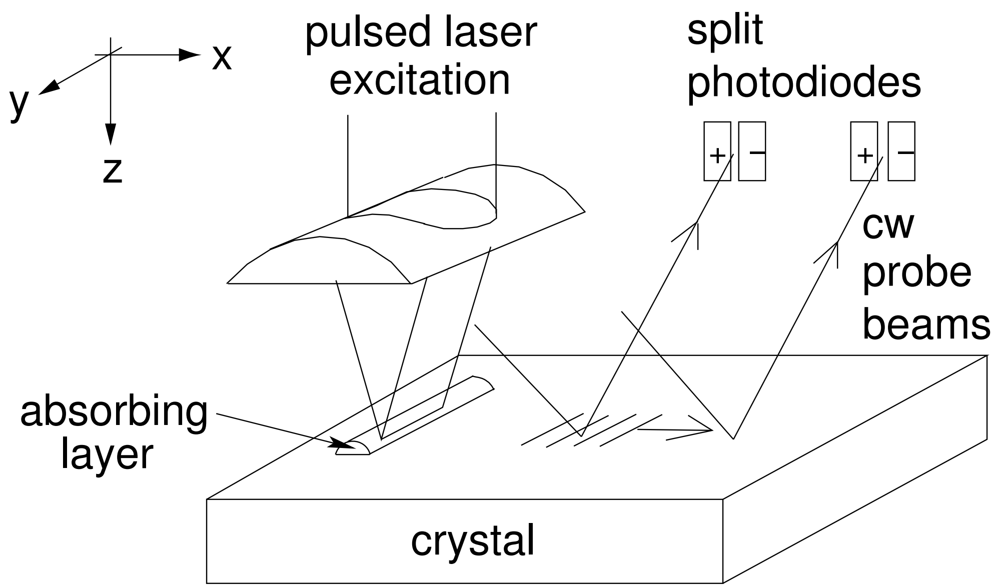
"Such curves all have mirror symmetry about the 30° direction and hexagonal symmetry in general on this surface."

Wave Speed:

Decay coefficients:

EXPERIMENT: NONLINEAR SAWS

Approach: Laser-excited, pressure-shock SAW generation [Lomonosov and Hess, 1996]



- Pulse detection:

  Probe beam deflection proportional to vertical vel.

  Temporal resolution: 0.4 to 1 ns

- Probe beam locations:

  1st beam: 5 mm from source

  2nd beam: 15 mm from 1st beam

- Resulting SAW pulses:

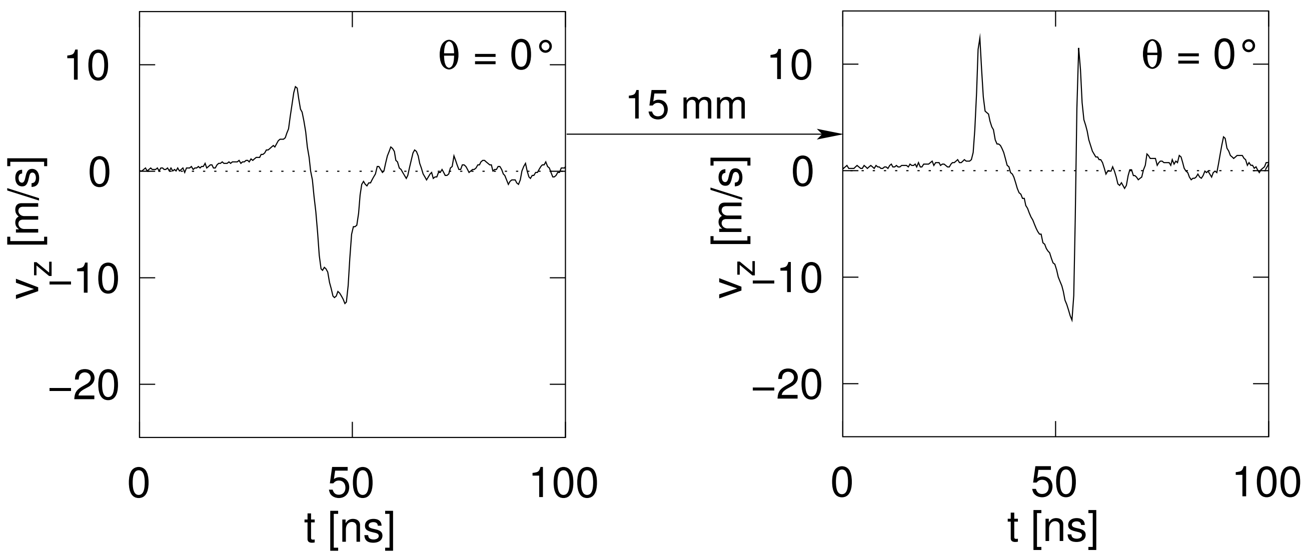
  Duration: 30 to 50 ns

  Peak strain: 0.002 to 0.004

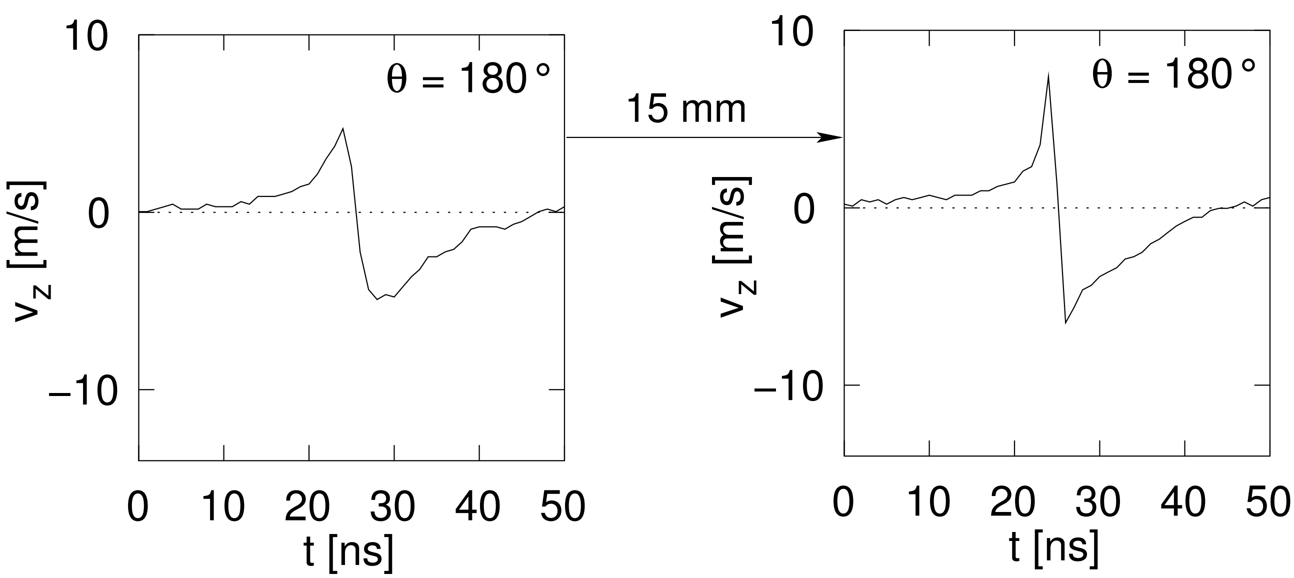
EXPERIMENTAL DATA

Vertical velocity waveforms for Si in (111) plane

Propagation direction: $\theta = 0^\circ$



Propagation direction: $\theta = 180^\circ$



Conclusion:
Waveform distortion does not have sixfold symmetry!

THEORY: NONLINEAR SAWS

Approach: Hamiltonian mechanics formalism
[Hamilton, Ilinskii, Zabolotskaya, 1996]

Nonlinear stress–strain relation:

$$\sigma_{ij} = c_{ijkl}e_{kl} + d_{ijklmn}e_{kl}e_{mn}$$

$c_{ijkl} \rightarrow$ Second order elastic constants

$d_{ijklmn} \rightarrow$ Third order elastic constants

Velocity waveforms in solid:

$$v_j(x,z,t) = \sum_{n=-\infty}^{\infty} v_n(x)u_{nj}(z)e^{ink(x-ct)}$$

$$u_{nj}(z) = \sum_{s=1}^{3} \beta_j^{(s)} e^{inkl_3^{(s)}z}$$

Coupled spectral evolution equations:

$$\frac{dv_n}{dx} + \alpha_n v_n = -\frac{n^2\omega c_{44}}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \widehat{S}_{lm} v_l v_m$$

$v_n \rightarrow$ nth harmonic amplitude

$\widehat{S}_{lm} \rightarrow$ nonlinearity matrix elements

$\alpha_n \rightarrow$ weak attenuation

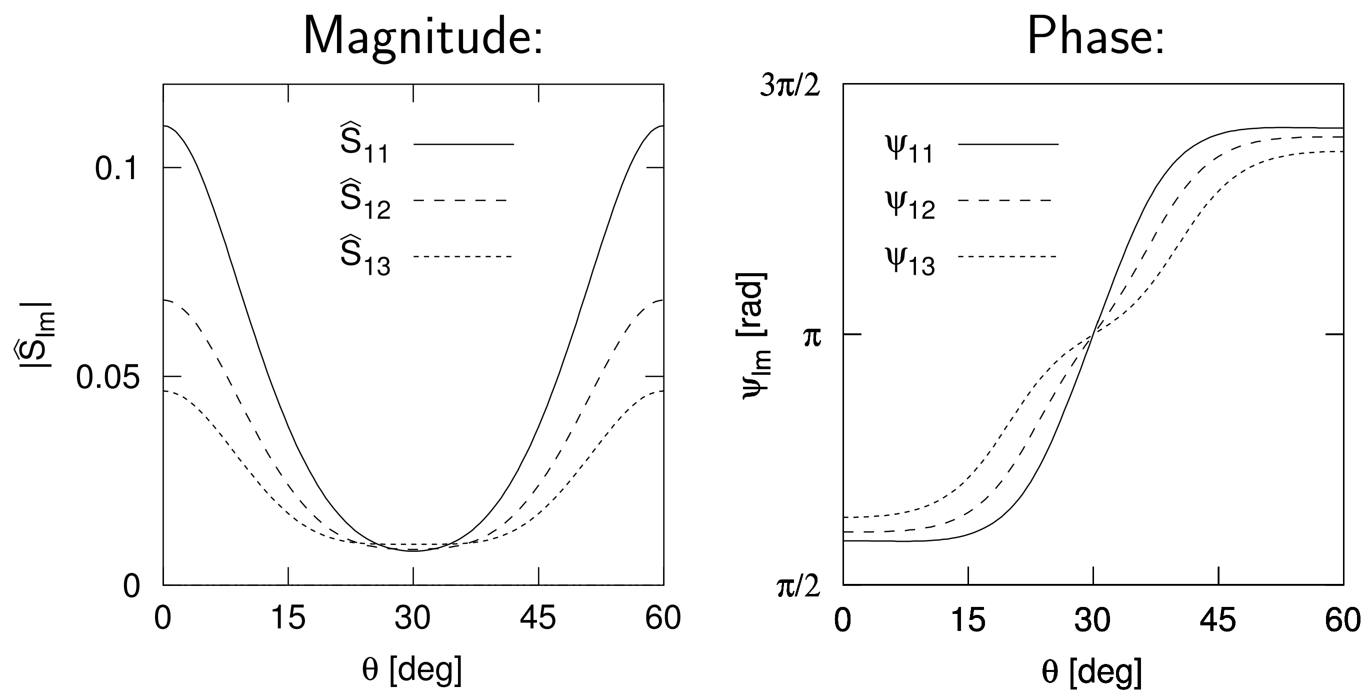
NONLINEARITY MATRIX

Nonlinearity matrix elements:

$$\widehat{S}_{lm} = \frac{-1}{c_{44}} \sum_{s_1,s_2,s_3=1}^{3} \frac{\frac{1}{2} d'_{ijpqrs} \beta_i^{(s_1)} \beta_p^{(s_2)} [\beta_r^{(s_3)}]^* l_j^{(s_1)} l_q^{(s_2)} [l_s^{(s_3)}]^*}{l l_3^{(s_1)} + m l_3^{(s_2)} - (l+m)[l_3^{(s_3)}]^*}$$

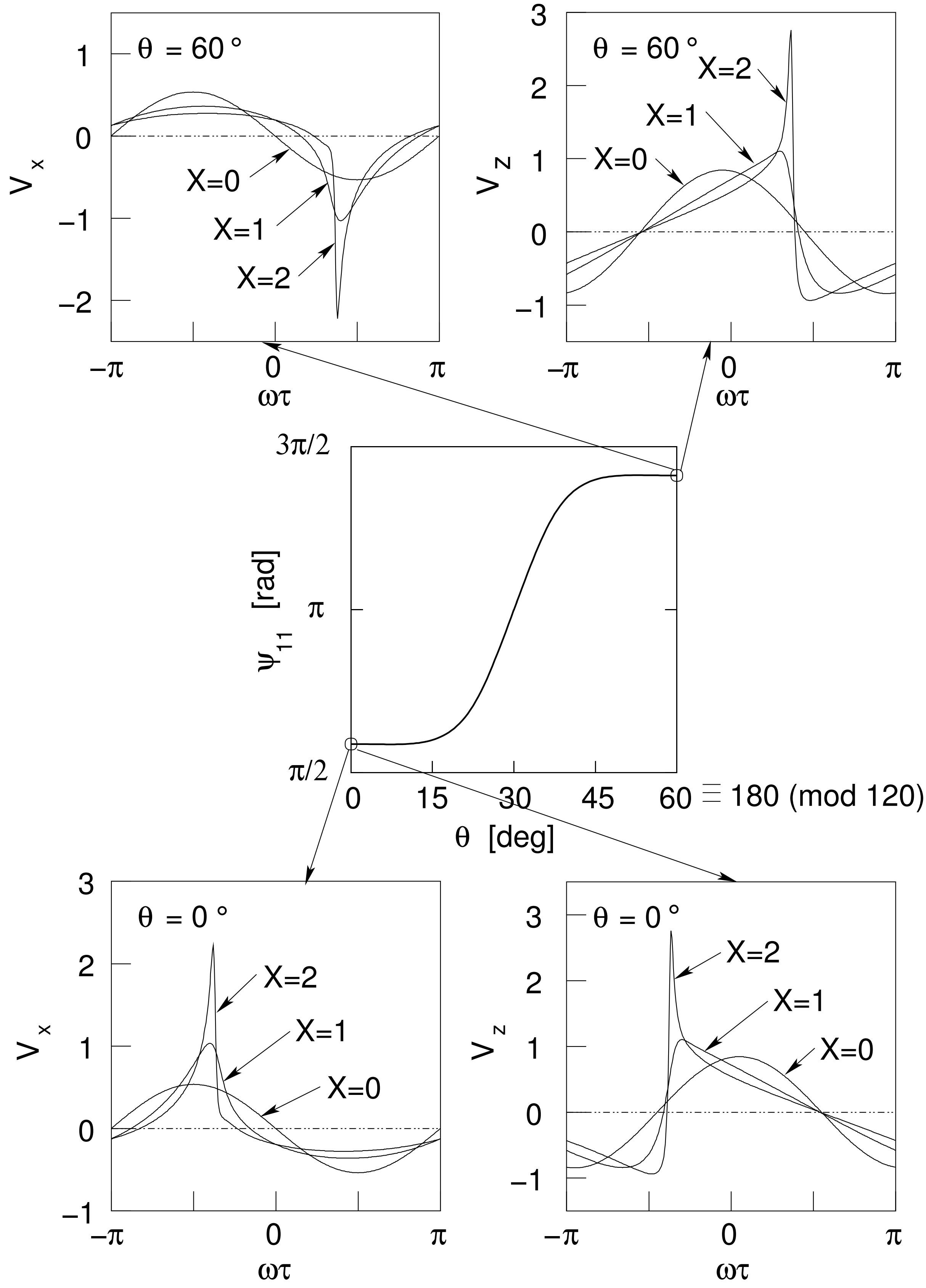
$$\begin{aligned} l_j^{(s)} &\rightarrow \text{depth penetration coefficients} \\ \beta_j^{(s)} &\rightarrow \text{amplitude coefficients} \\ d'_{ijpqrs} &\rightarrow \text{2nd and 3rd order elastic constants} \end{aligned}$$

Angular dependence of $\widehat{S}_{lm}$ in (111) cut of Si:

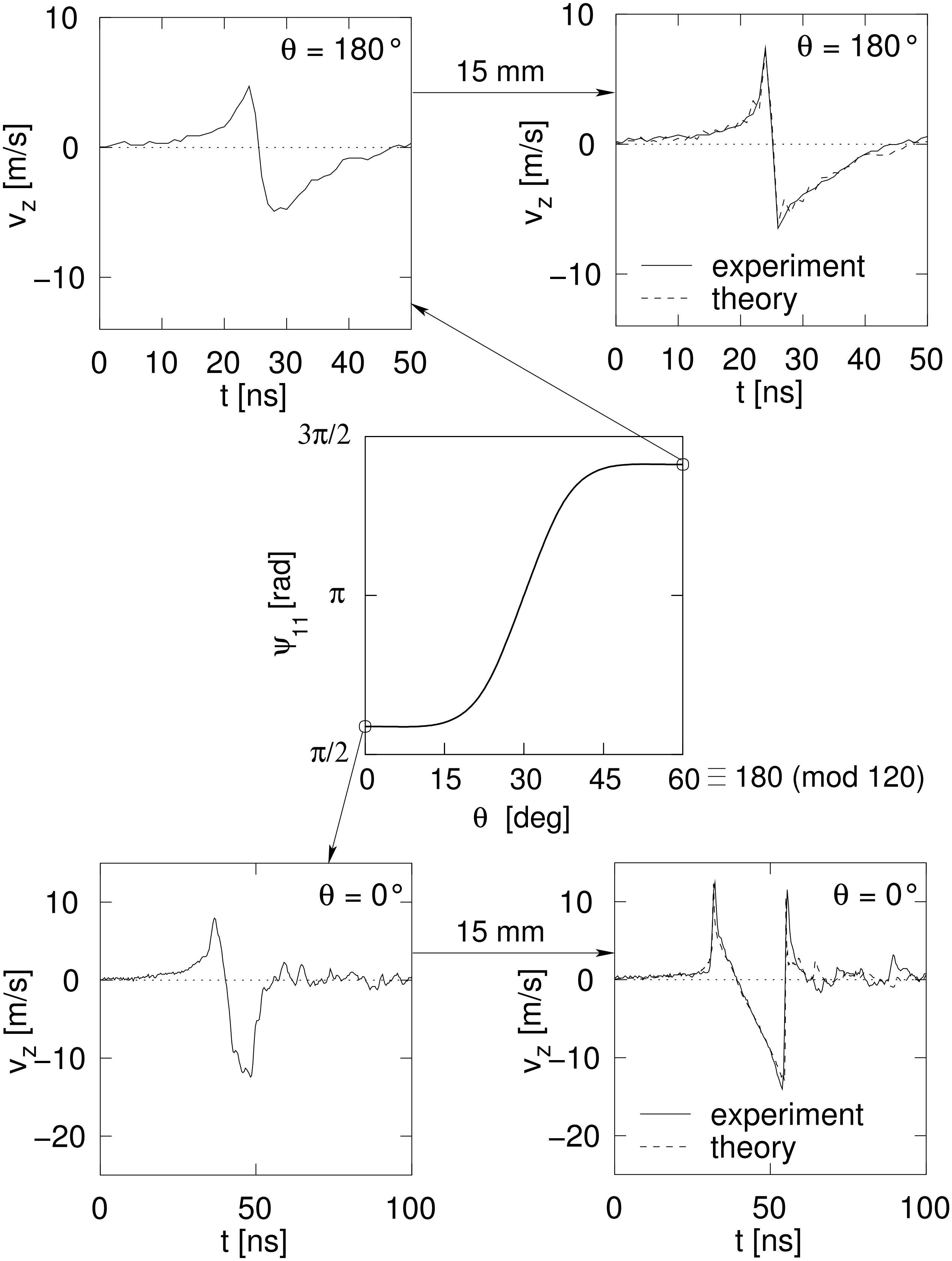


- Magnitude: *Periodic every 60°*
- Phase: *Periodic every 120°*

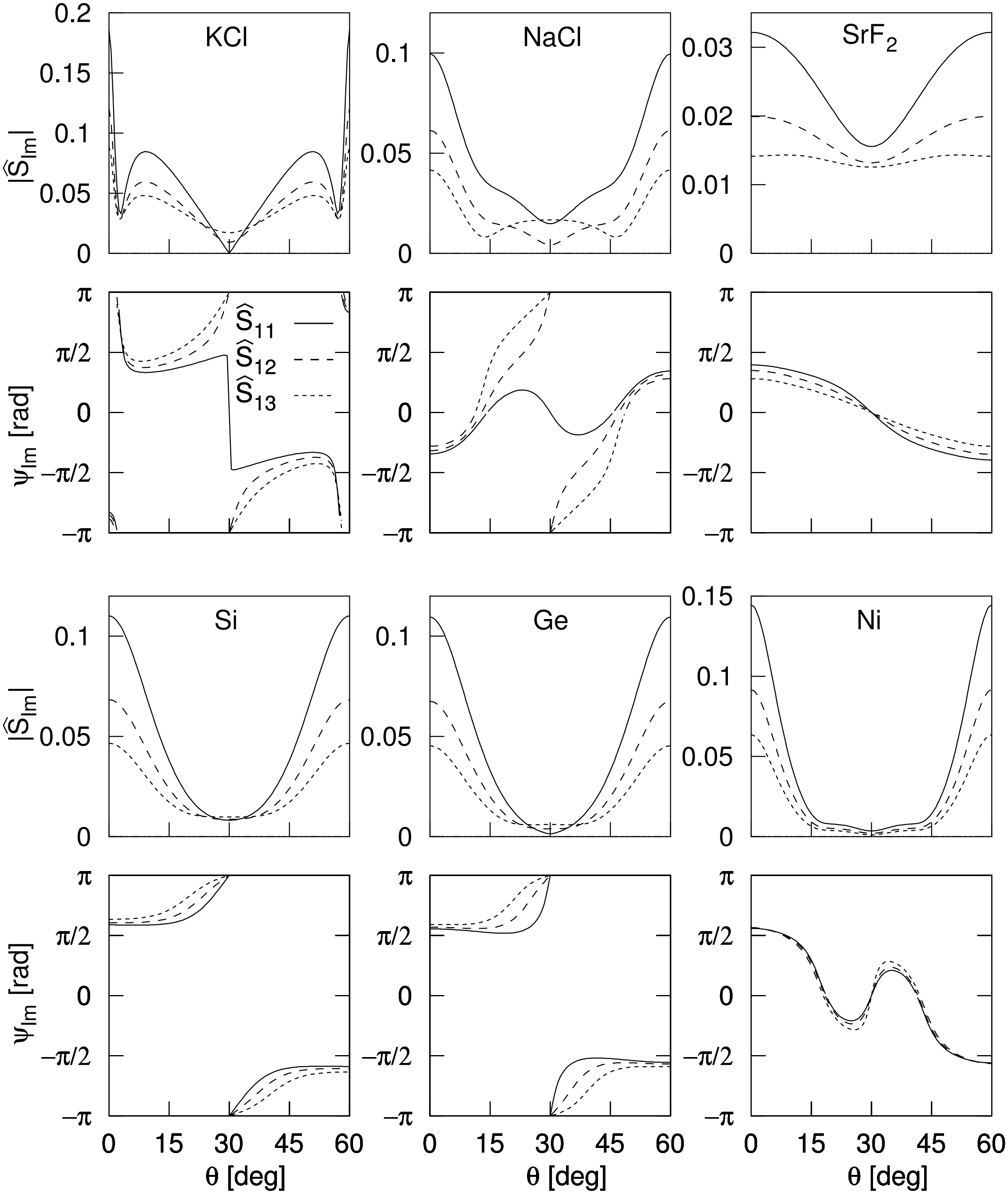
SINUSOID DISTORTION FOR SI IN (111) PLANE



COMPARISON OF THEORY AND EXPERIMENT



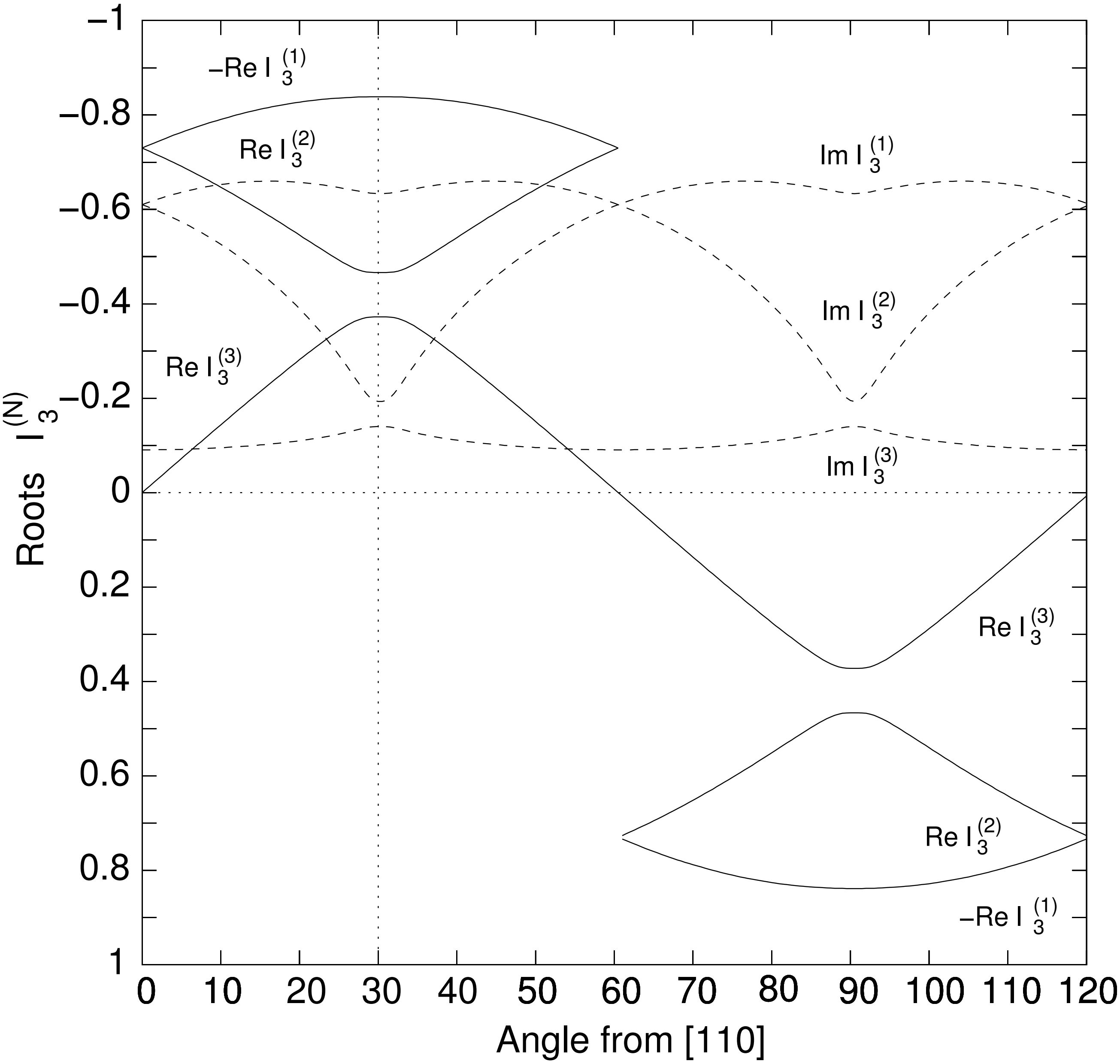
OTHER CUBIC CRYSTALS



PERIODICITY OF LINEAR PARAMETERS (REVISITED)

KCl in (111) plane over angular range $0^\circ \leq \theta \leq 120^\circ$

Depth penetration coefficients:



CONCLUSIONS

- Measurements of nonlinear surface waves in (111) plane of crystalline silicon indicate that waveform distortion exhibits threefold symmetry.

- Nonlinearity matrix elements describing harmonic generation have sixfold symmetry in magnitude, but only threefold symmetry in phase.

- Calculated waveform distortion is corroborated by measured waveforms.

- Threefold symmetry of nonlinearity is also predicted for other cubic crystals besides silicon.

- Certain linear surface wave parameters in the (111) plane (e.g., the depth penetration coefficients) have only threefold symmetry, in contrast to others which have sixfold symmetry (e.g., wave speed).