NONLINEAR SURFACE ACOUSTIC WAVES IN CUBIC CRYSTALS

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SUMMARY OF RESULTS

- A nonlinearity matrix that describes the coupling between harmonics is shown to provide a useful tool for characterizing waveform distortion in different crystals, surface cuts, and propagation directions.

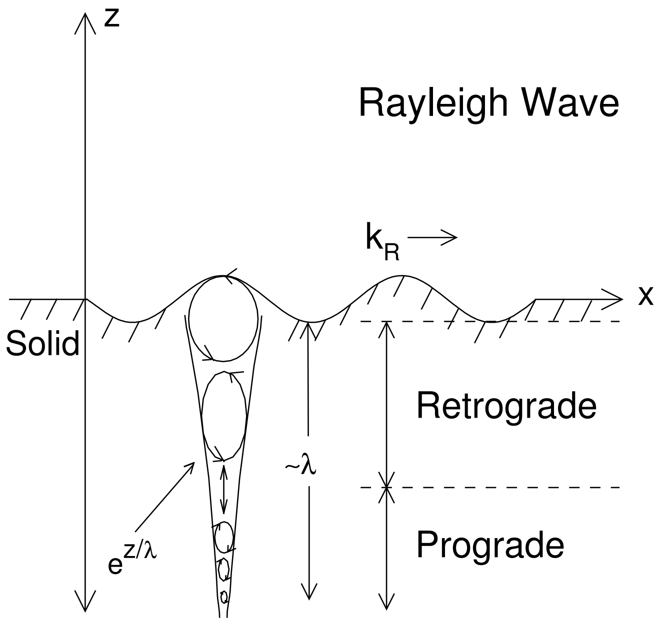
- In the (001) surface cut:

  - Distortion type depends sensitively on direction.
  - Example: longitudinal velocity component varies between compression and rarefaction shocks.
  - Calculated waveforms corroborated by measured data in crystalline silicon.

- In the (111) surface cut:

  - Initially sinusoidal waveforms exhibit asymmetric distortion as they propagate.
  - Harmonic phase plays an important role in the distortion process.
  - Calculated waveforms are corroborated by measured data in crystalline silicon.

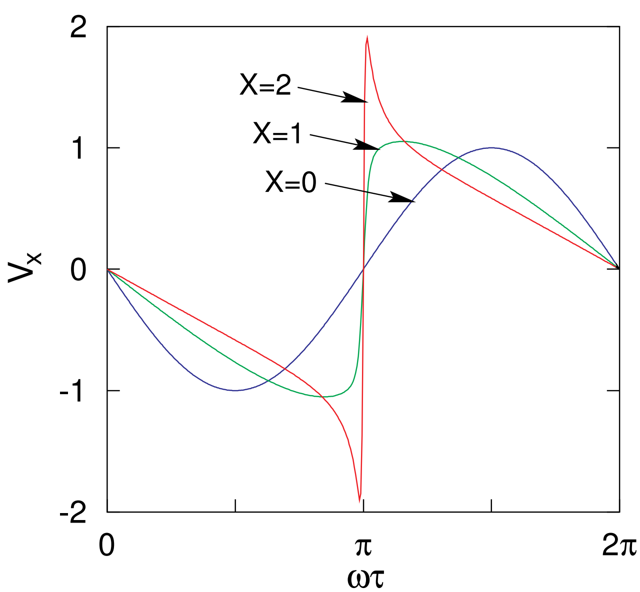
NONLINEAR SURFACE WAVES

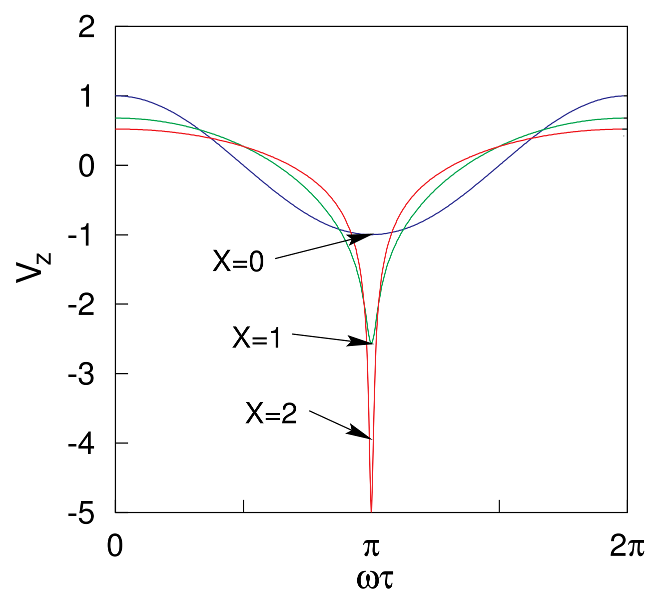


Surface waves are good candidates for shock formation because:

- Energy is concentrated near the surface.
- All harmonics travel at the same wave speed (i.e., nondispersive).

Waveform distortion in isotropic media:





- Waveforms are "snapshots" in frame moving at wave speed.
- Propagation distance X scaled by shock formation distance.

NONLINEAR THEORY

[M. F. Hamilton, Yu. A. Il'inskii, and E. A. Zabolotskaya, J. Acoust. Soc. Am. **105**, 639–651 (1999).]

Velocity waveforms in solid:

$$v_j(x,z,t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)}$$

$$\begin{aligned} v_n(x) &\rightarrow n\text{th harmonic amplitude} \\ u_{nj}(z) &\rightarrow \text{depth functions from linear solution} \end{aligned}$$

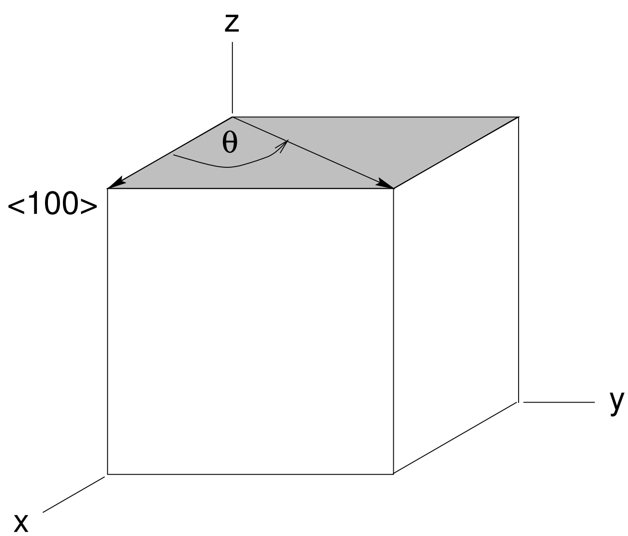
Coupled spectral evolution equations:

$$\frac{dv_n}{dx} + \alpha_n v_n = -\frac{n^2 \omega c_{44}}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \widehat{S}_{lm} v_l v_m$$

The nonlinearity matrix elements $\widehat{S}_{lm}$ are:

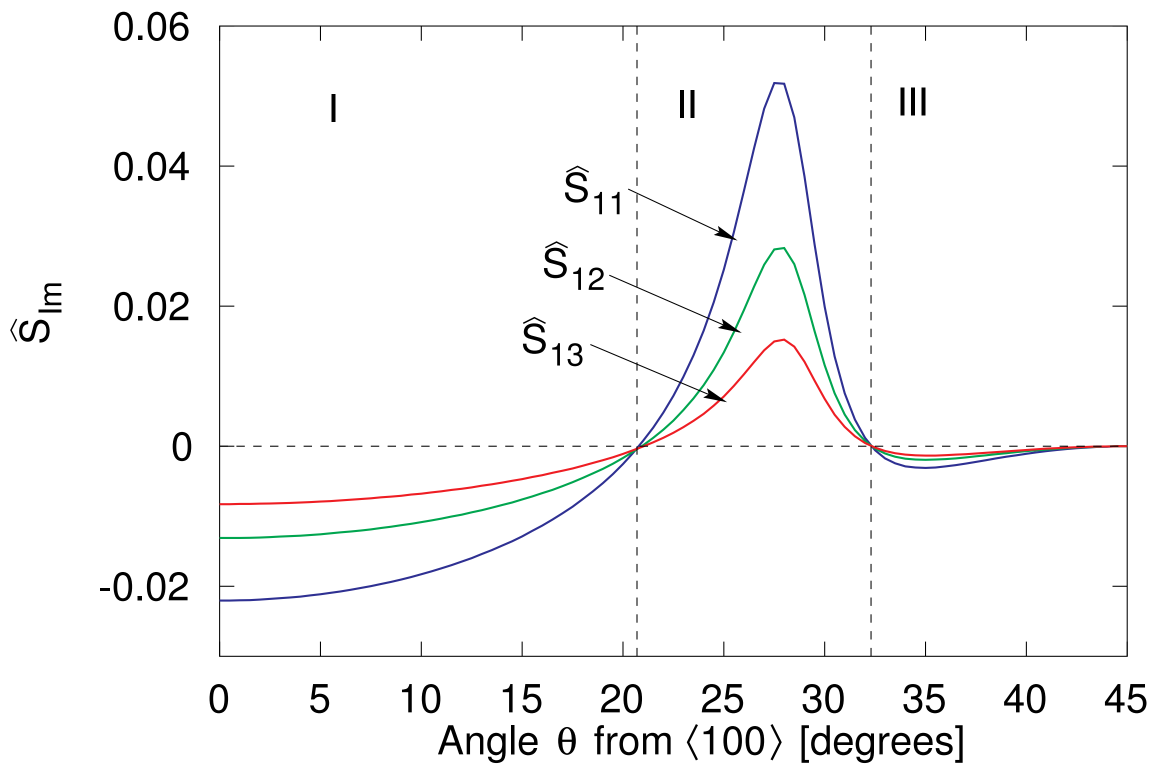
- coefficients which represent coupling of lth and mth harmonics to generate $n = (l+m)$th harmonic,
- only a function of density, 2nd and 3rd order elastic constants, and the solution of the linear problem,
- useful for mapping the types of waveform distortion as a function of propagation direction.

REAL-VALUED NONLINEARITY: (001) CUT



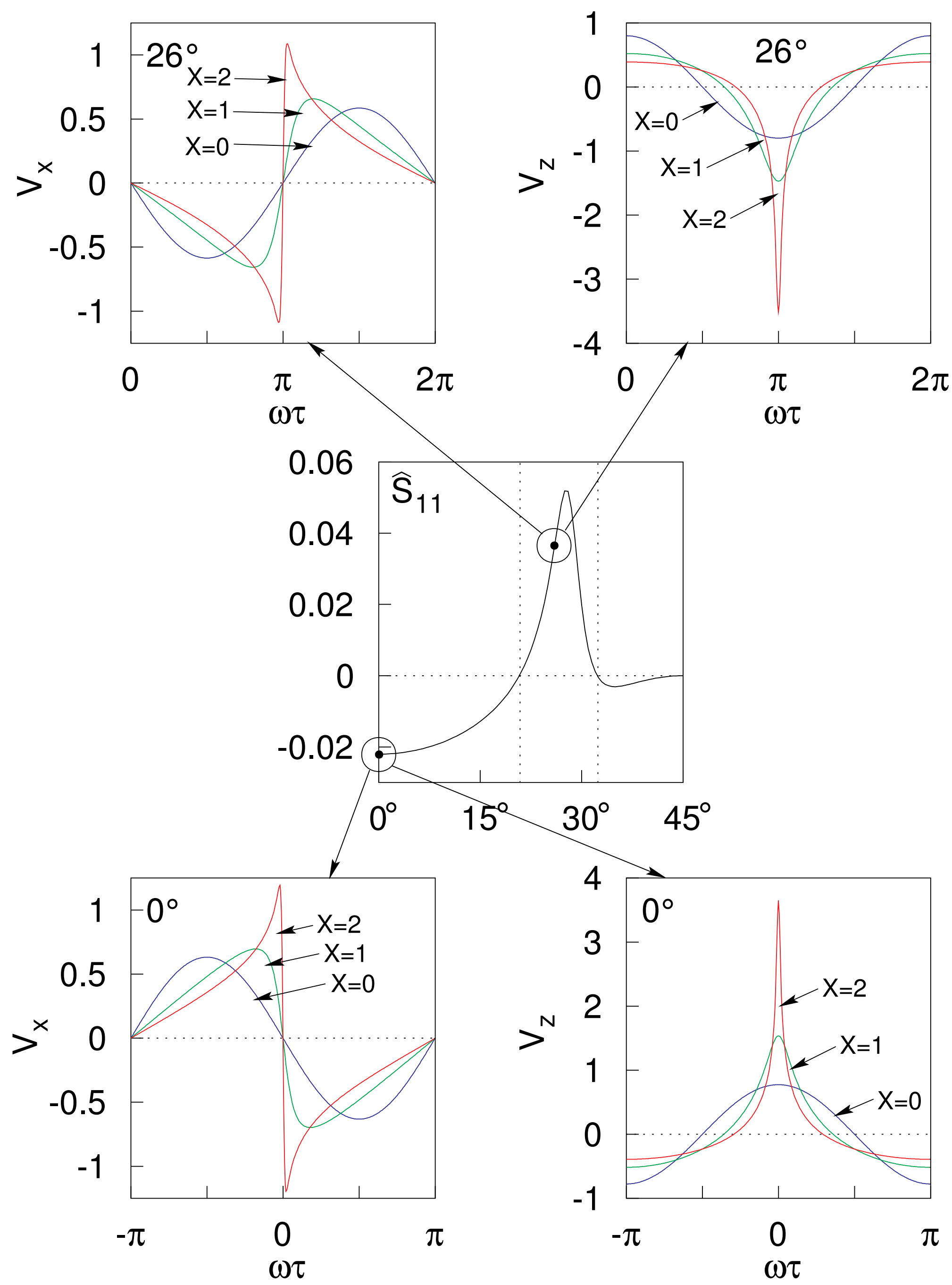
- Plane of mirror symmetry.
- All $\widehat{S}_{lm}$ are real-valued.

Angular dependence of $\widehat{S}_{lm}$ in Si:



- Surface waves exhibit 3 distinct regions of nonlinearity with corresponding variation in waveform distortion.

SIMULATIONS WITH SINUSOIDS: SI (001)

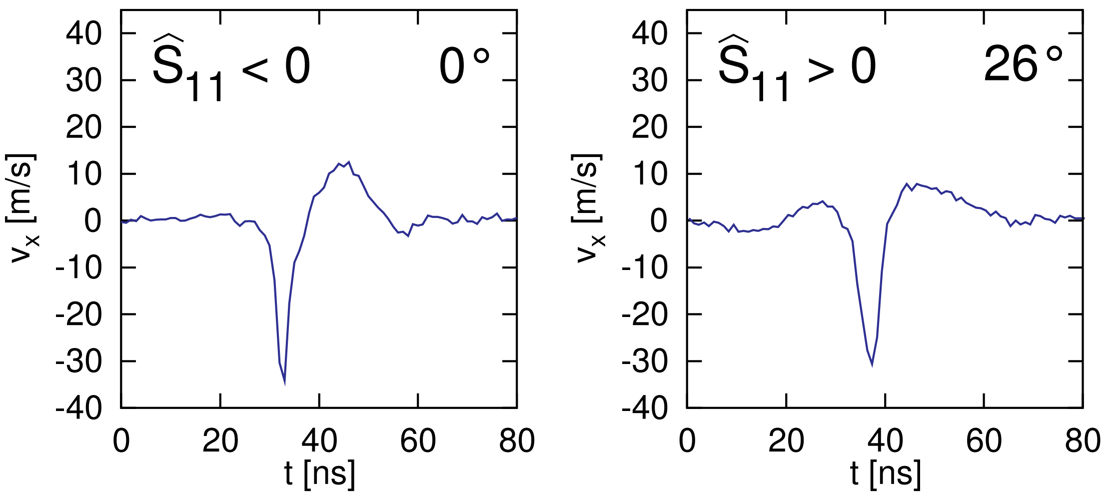


- Top waveforms show positive nonlinearity (compression shock in V_x) in Region II.
- Bottom waveforms show negative nonlinearity (rarefaction shock in V_x) in Region I.

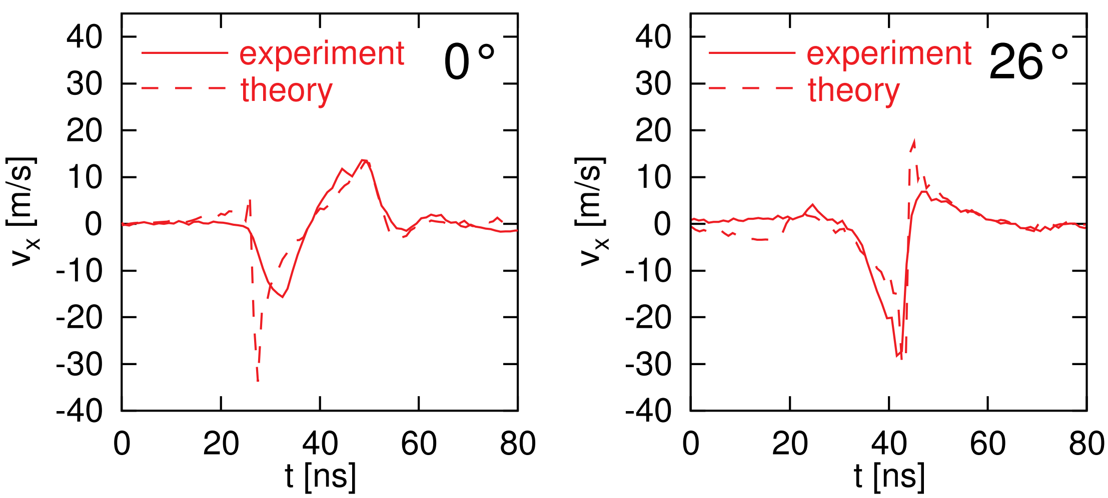
EXPERIMENT: SI (001), 0° and 26° from ⟨100⟩

Method: Laser surface wave generation and detection
[from A. Lomonosov and P. Hess, University of Heidelberg]

Longitudinal velocity component, 5 mm from source:

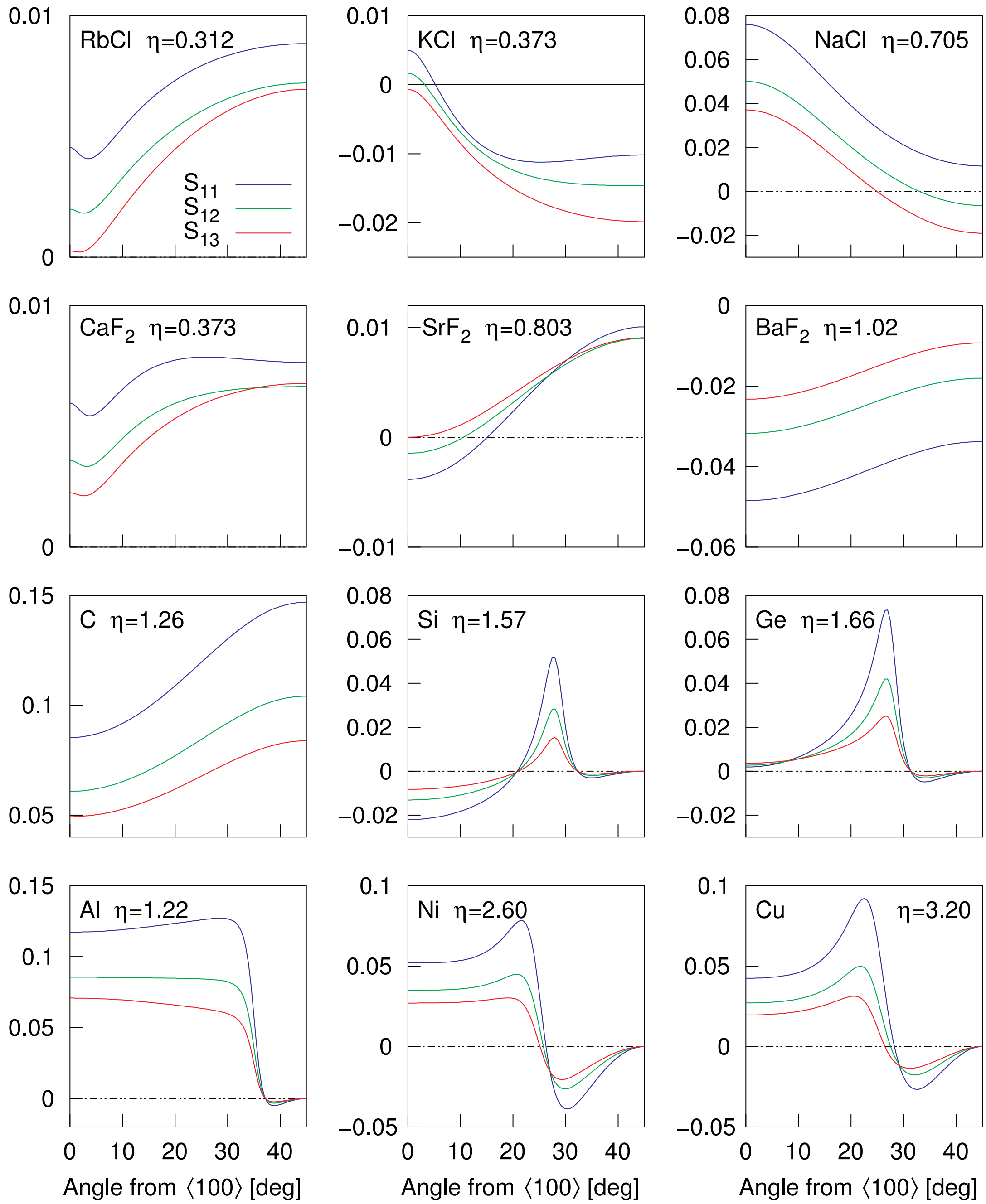


Same pulses, 20 mm from source:



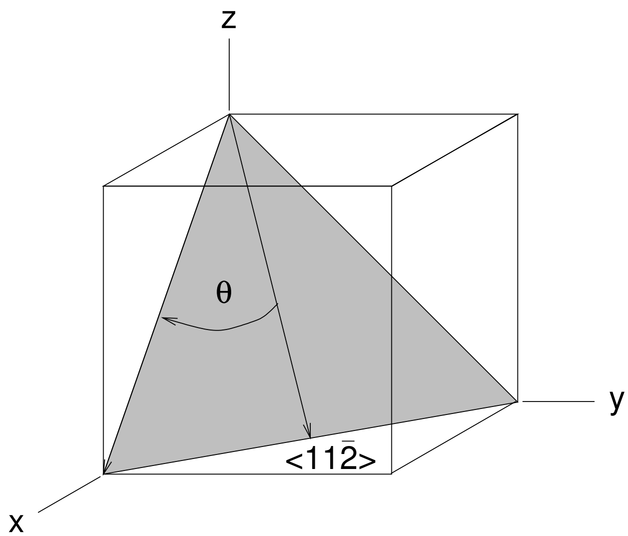
- Rarefaction shock forms at $\theta = 0^\circ$, while compression shock forms at $\theta = 26^\circ$.
- Waveforms are similar at x=5 mm, but quite different at x=20 mm.

NONLINEARITY MATRICES: (001) PLANE



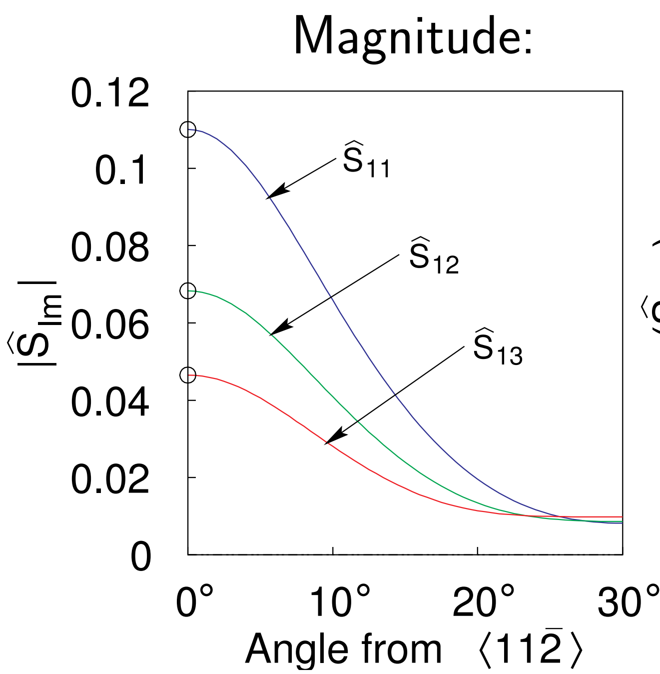
- Note diversity of $\widehat{S}_{lm}$ for a variety of crystals.
- Plots are ordered by the anisotropy ratio $\eta = c_{44}/(c_{11} - c_{12})$. For isotropic media, $\eta = 1$.

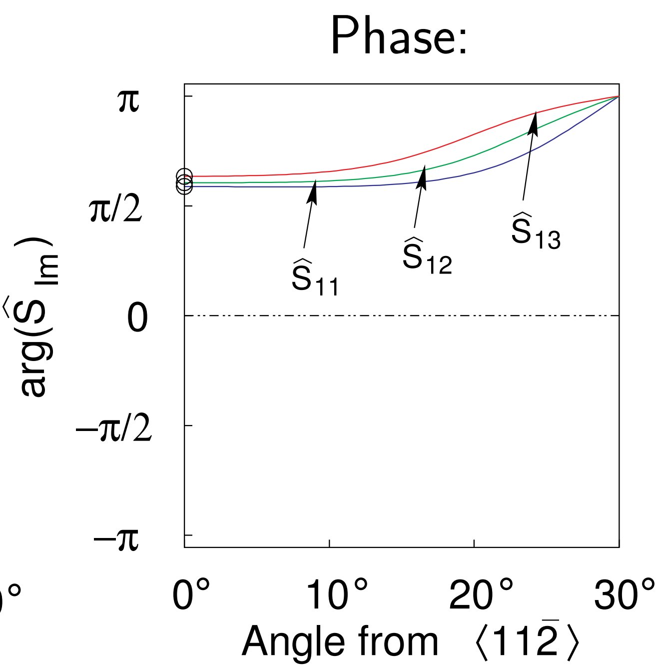
COMPLEX-VALUED NONLINEARITY: (111) CUT



- $\widehat{S}_{lm}$ are complex-valued in most directions.

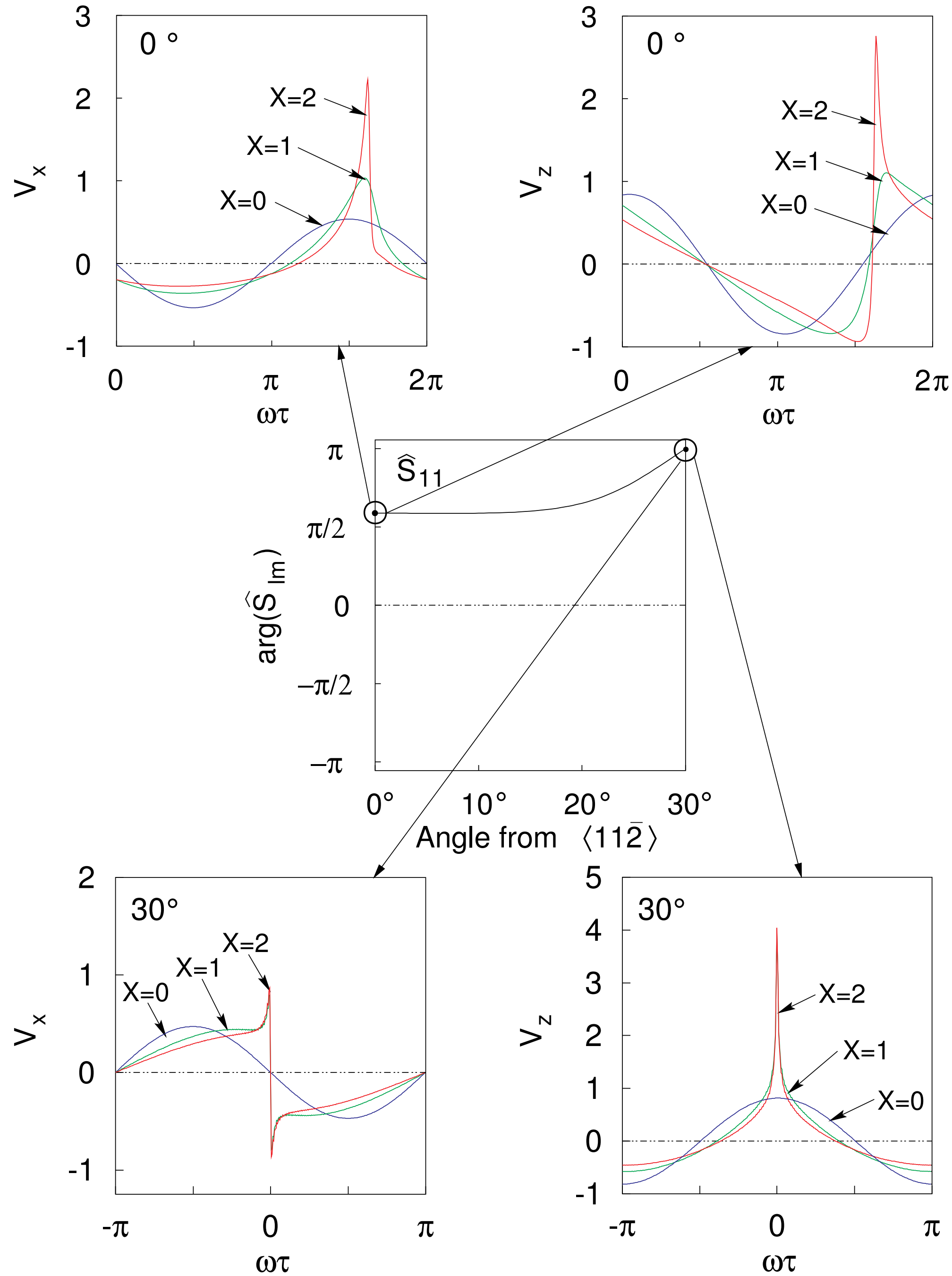
Angular dependence of $\widehat{S}_{lm}$ in Si:





- $\arg(\widehat{S}_{lm}) = 0$ corresponds to positive nonlinearity.
- $\arg(\widehat{S}_{lm}) = \pi$ corresponds to negative nonlinearity.

SIMULATIONS WITH SINUSOIDS: SI (111)

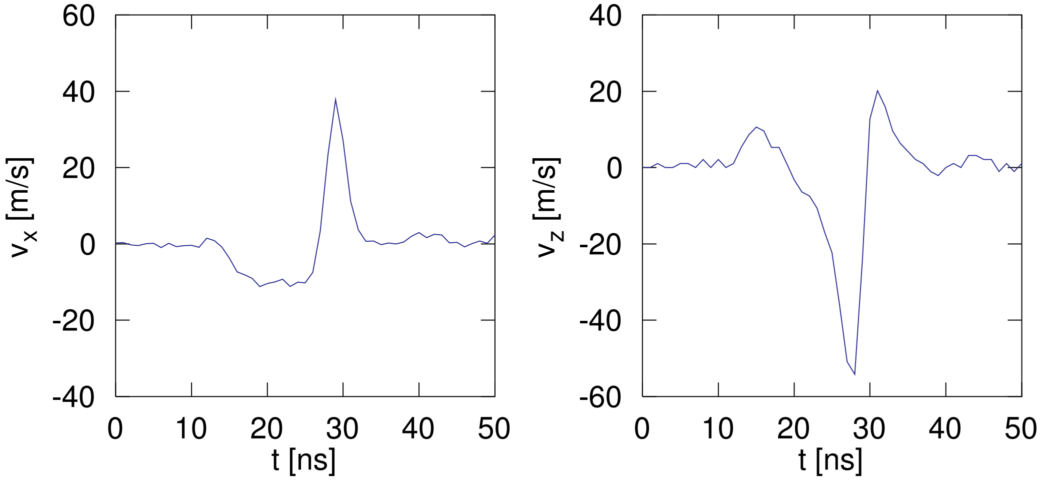


- Top waveforms show complex-valued $\widehat{S}_{lm}$ at $\theta = 0^\circ$ result in asymmetric distortion.
- Bottom waveforms show negative, real-valued $\widehat{S}_{lm}$ at $\theta = 30^\circ$ result in symmetric distortion.

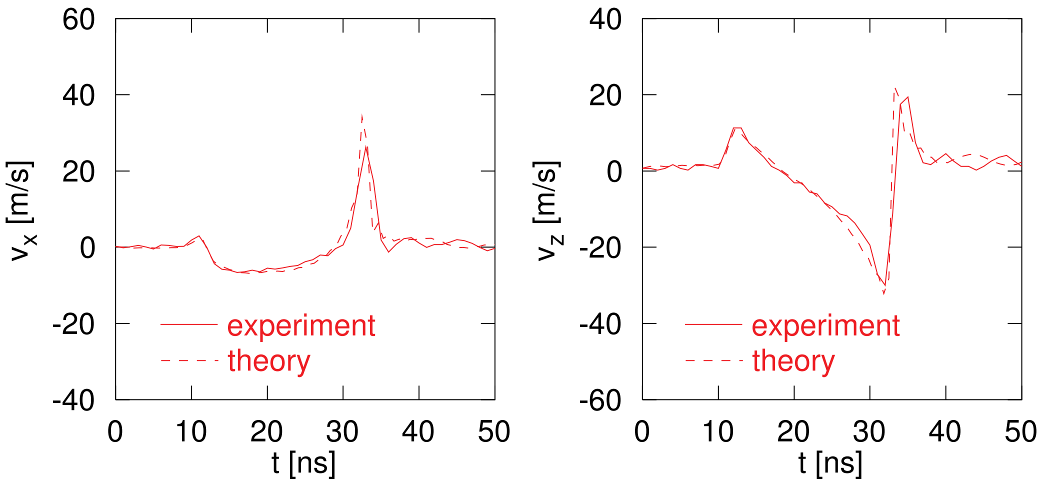
EXPERIMENT: SI (111), 0° from $\langle 11\bar{2} \rangle$

Method: Laser surface wave generation and detection
[from A. Lomonosov and P. Hess, University of Heidelberg]

Velocity waveforms, 5 mm from source:

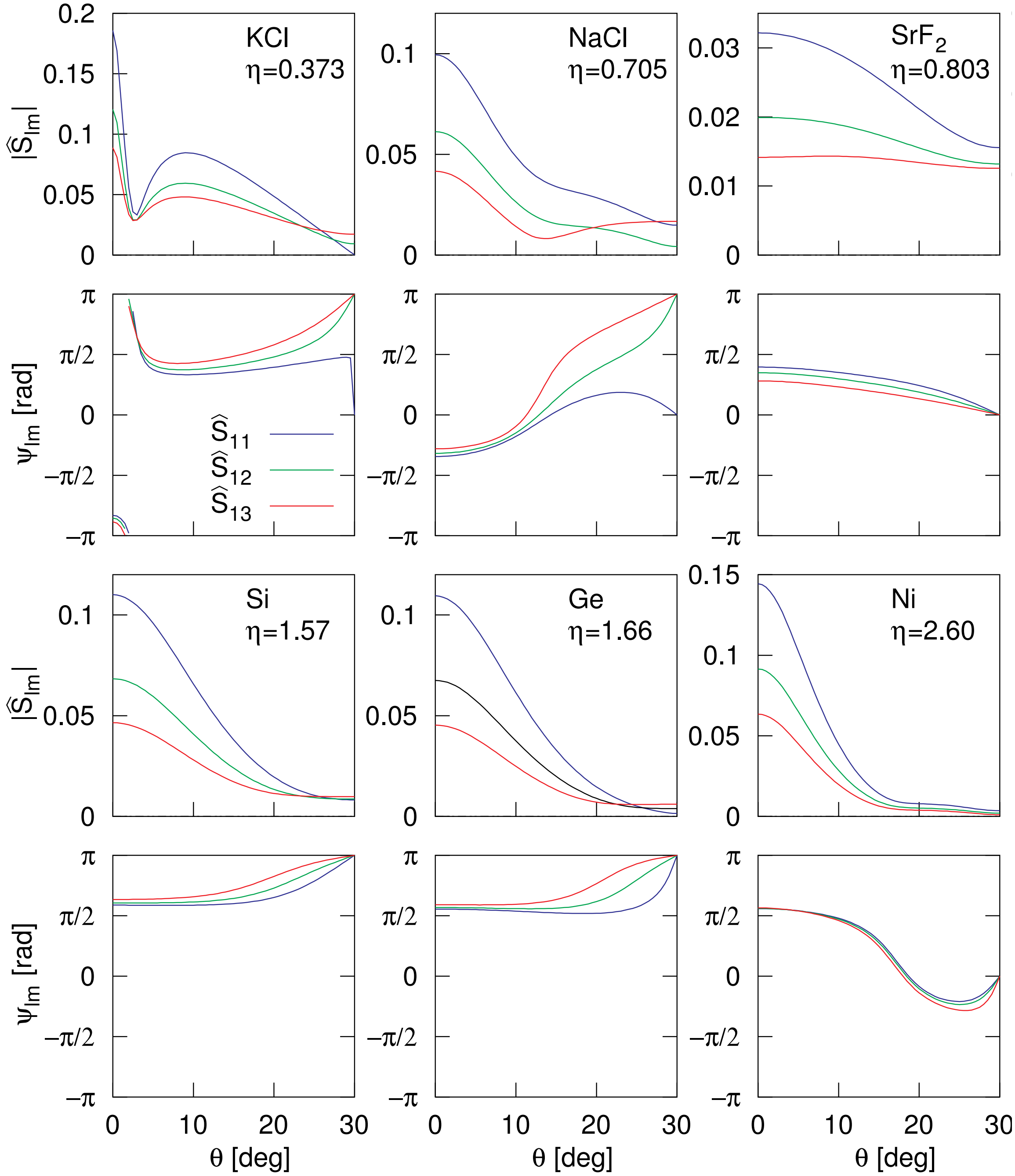


Same pulses, 21 mm from source:



- Waveforms distort similarly to the sinusoids (v_x distorts in U-shape; v_z, in N-shape).
- Note v_x is differs from (001) cut even with the same excitation process.

NONLINEARITY MATRICES: (111) PLANE



- $\widehat{S}_{lm}$ for a variety of crystals, ordered by anisotropic ratio.
- Although the magnitudes show similar trends, the phases vary from case to case $[\psi_{lm} = \arg(\widehat{S}_{lm})]$.