NONLINEAR SURFACE ACOUSTIC WAVES IN CUBIC CRYSTALS

Ronald E. Kumon* **and Mark F. Hamilton**

Department of Mechanical Engineering

The University of Texas at Austin

Austin, Texas, U.S.A.

*currently at the

National Institute of Standards and Technology

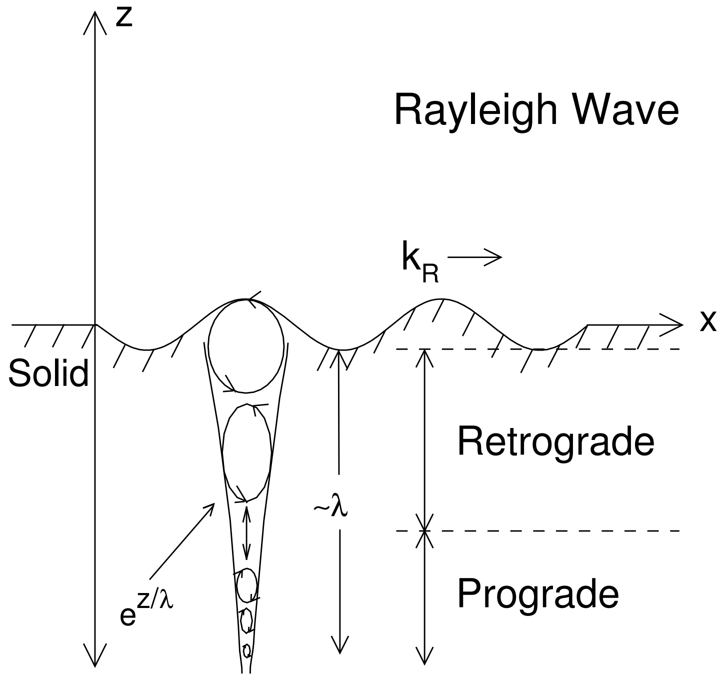
Boulder, Colorado, U.S.A.

OUTLINE

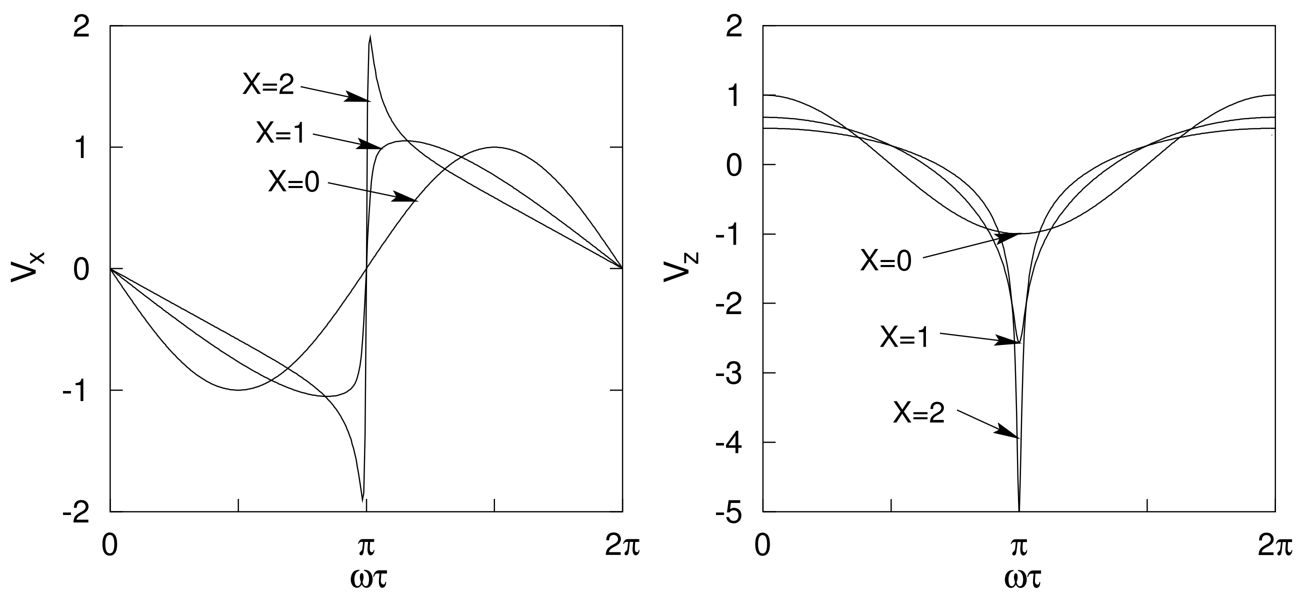
- Background
  - Nonlinear surface waves
  - Surface waves in crystals
  - Nonlinear theory
- Results in (001) plane
  - Simulations
  - Experiment
- Results in (111) plane
  - Simulations
  - Experiment
  - Approximate Method
- Summary

NONLINEAR SURFACE WAVES

Schematic Diagram:

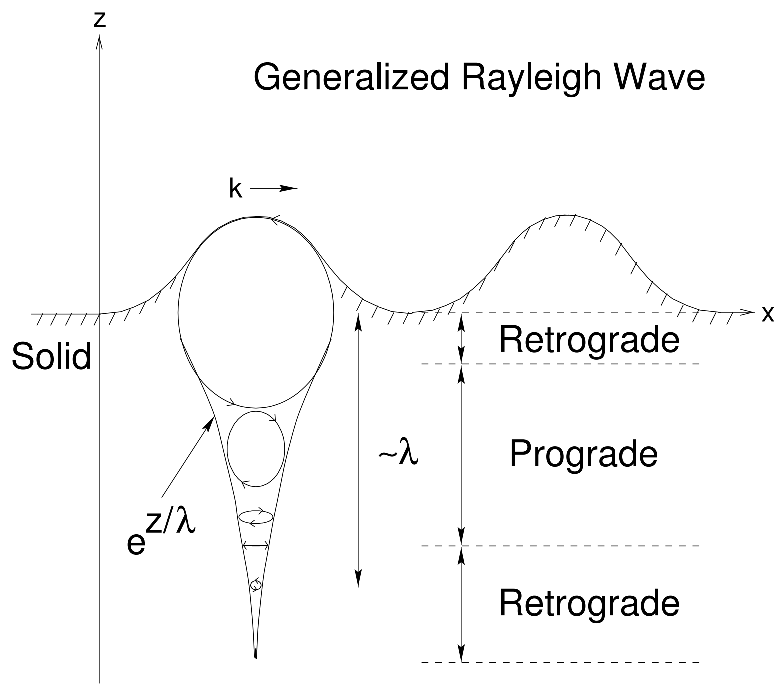


Waveform Distortion:
(in isotropic media or certain mirror planes of cubic crystals)

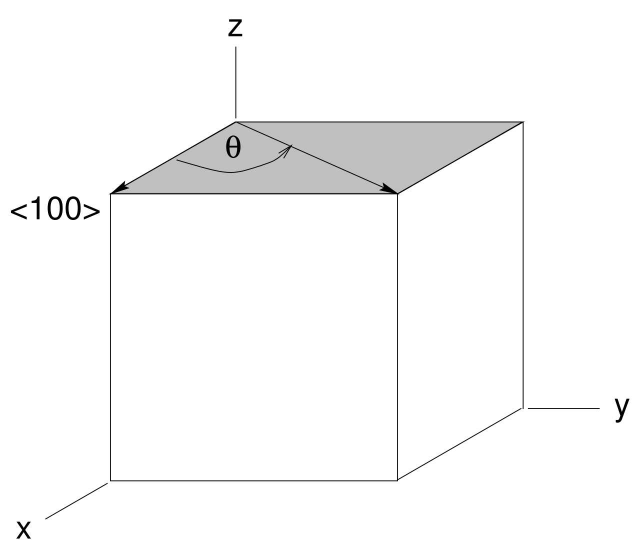


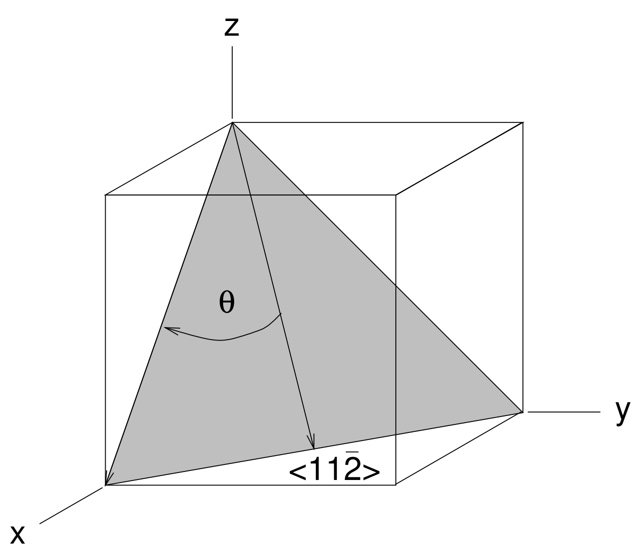
SURFACE WAVES IN CRYSTALS

Schematic Diagram:



Typical Planes for Crystal Cuts:





NONLINEAR THEORY

Approach: Hamiltonian mechanics formalism
(Hamilton, Il'inskii, Zabolotskaya, 1996)

Velocity waveforms in solid:

$$v_j(x,z,t) = \sum_{n=-\infty}^{\infty} v_n(x) u_{nj}(z) e^{in(kx-\omega t)}$$

$$u_{nj}(z) = \sum_{s=1}^{3} \beta_j^{(s)} e^{ink\lambda_3^{(s)} z}$$

Coupled spectral evolution equations:

$$\frac{dv_n}{dx} + \alpha_n v_n = -\frac{n^2 \omega c_{44}}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \widehat{S}_{lm} v_l v_m$$

$v_n \rightarrow$ nth harmonic amplitude
$\widehat{S}_{lm} \rightarrow$ nonlinearity matrix elements
$\alpha_n \rightarrow$ weak attenuation

Observation:

(001) plane $\Rightarrow$ S_{lm} real valued
(111) plane $\Rightarrow$ S_{lm} complex valued

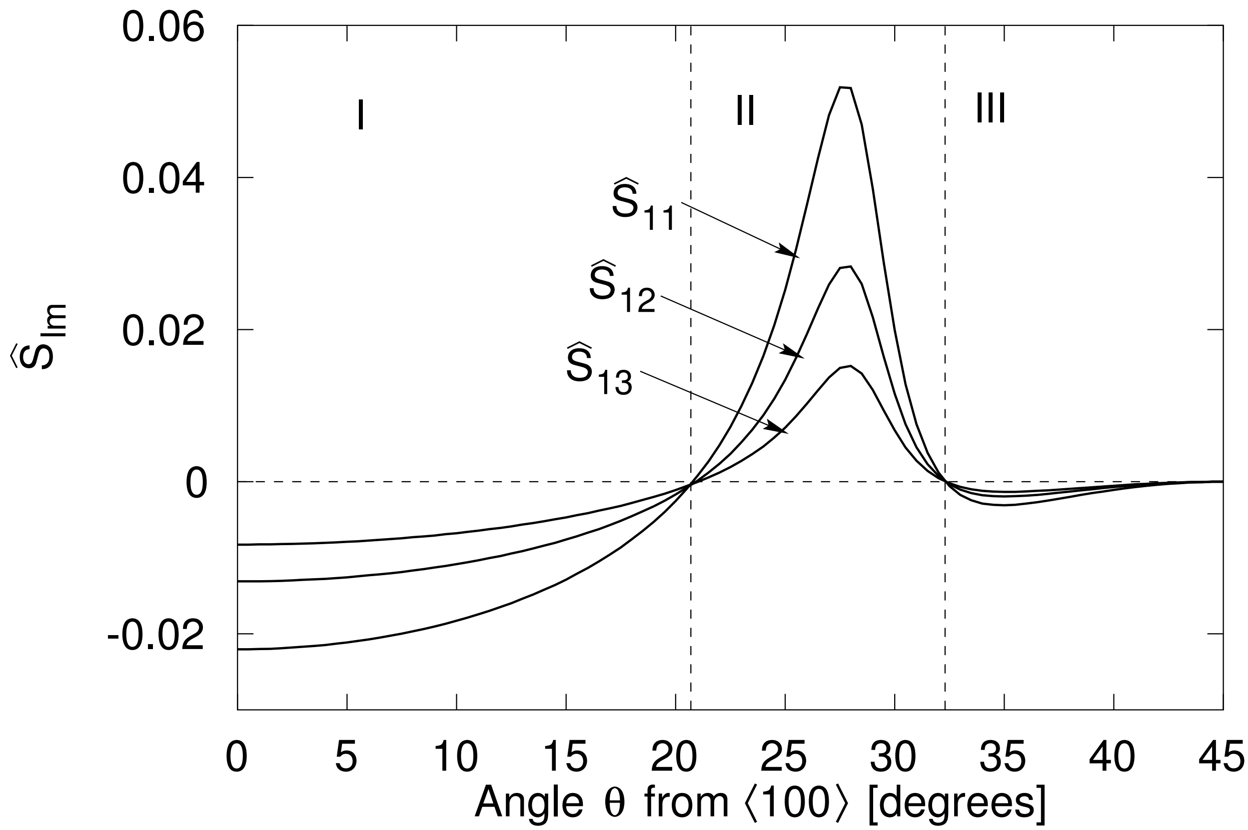
NONLINEARITY MATRIX FOR SI (001)

Nonlinearity matrix elements:

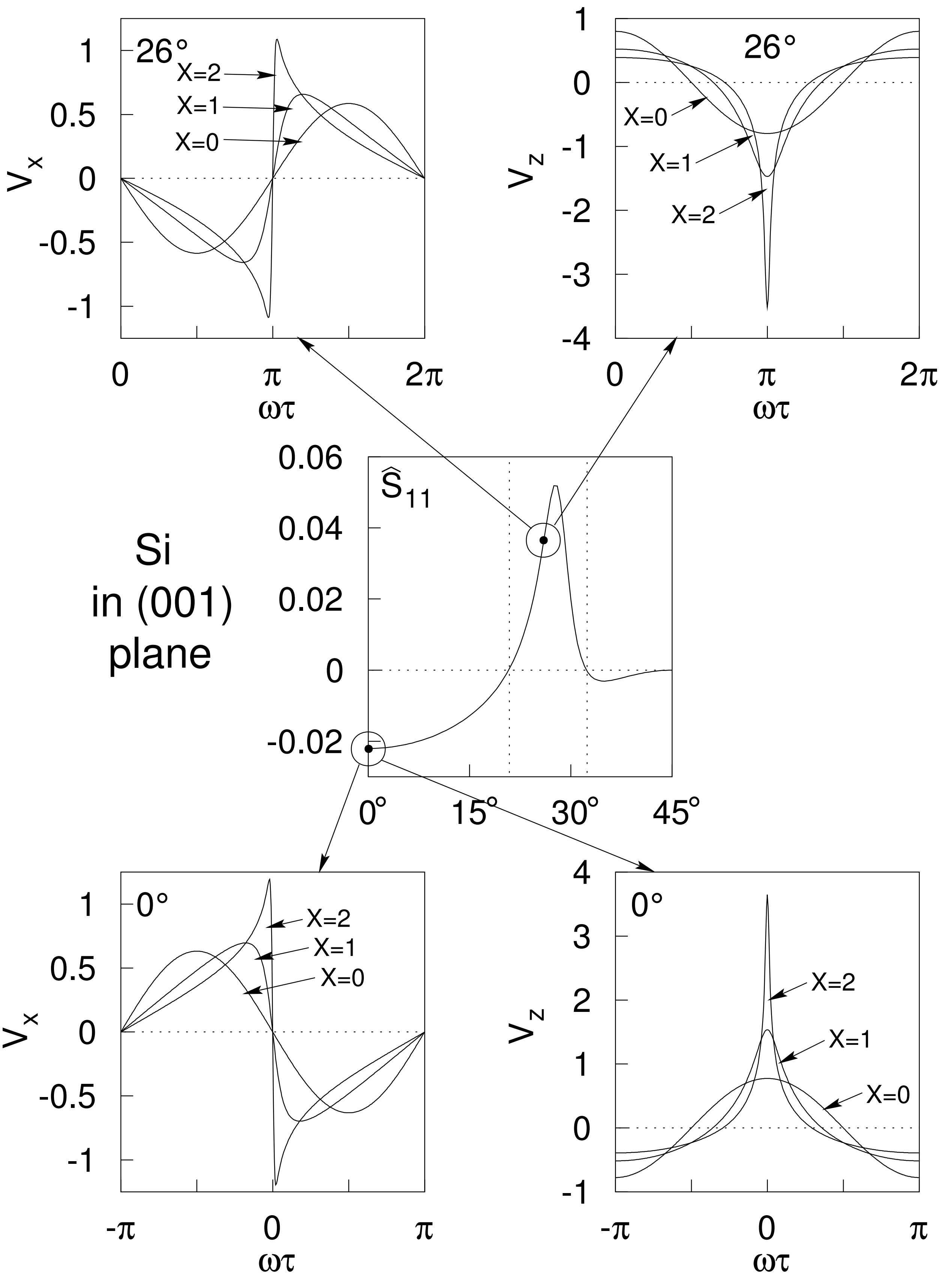
$$\widehat{S}_{lm} = \frac{-1}{c_{44}} \sum_{s_1,s_2,s_3=1}^{3} \frac{\frac{1}{2} d'_{ijpqrs} \beta_i^{(s_1)} \beta_p^{(s_2)} [\beta_r^{(s_3)}]^* \lambda_j^{(s_1)} \lambda_q^{(s_2)} [\lambda_s^{(s_3)}]^*}{l\lambda_3^{(s_1)} + m\lambda_3^{(s_2)} - (l+m)[\lambda_3^{(s_3)}]^*}$$

$\lambda_i^{(s)} \rightarrow$ eigenvalues of linear problem
$\beta_j^{(s)} \rightarrow$ eigenvectors of linear problem
$d'_{ijpqrs} \rightarrow$ 2nd and 3rd order elastic constants

Selected matrix elements for Si in (001) plane:

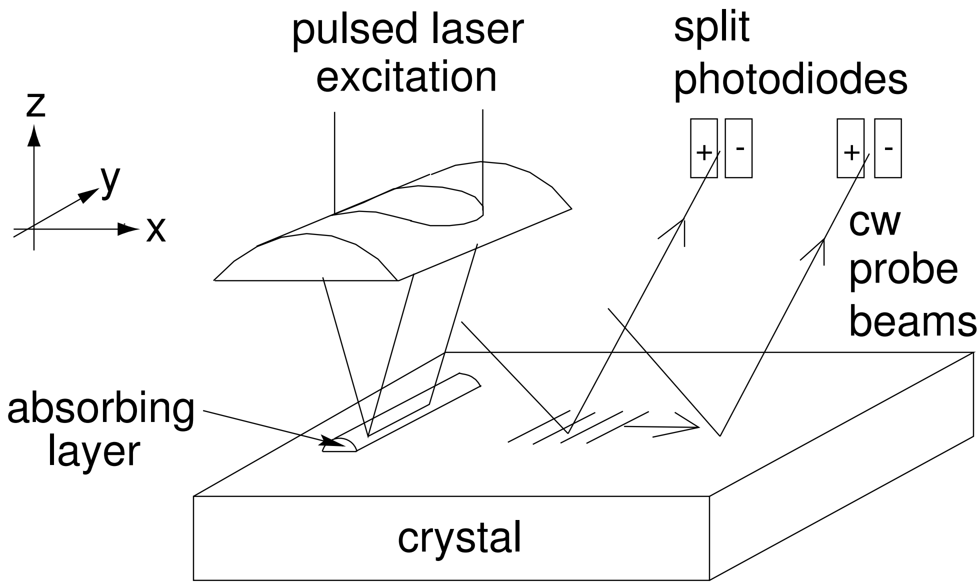


SIMULATIONS WITH SINUSOIDS FOR SI (001)



EXPERIMENT

Approach: Laser-excited photoelastic SAW generation (Lomonosov and Hess, 1996)



- Pulse detection:

  Probe beam deflection proportional to vertical vel.

  Temporal resolution: 1 ns

- Beam locations:

  1st probe beam: 5 mm from source

  2nd probe beam: 10–15 mm from 1st probe beam

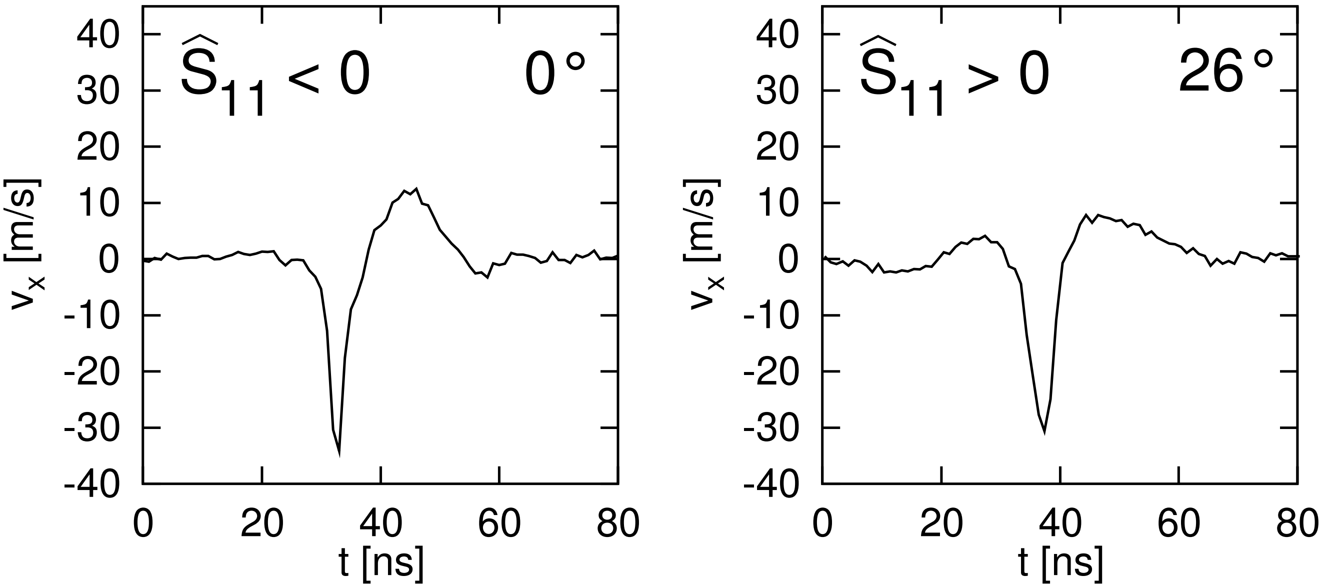
- Resulting SAW pulses:

  Duration: 20 to 40 ns

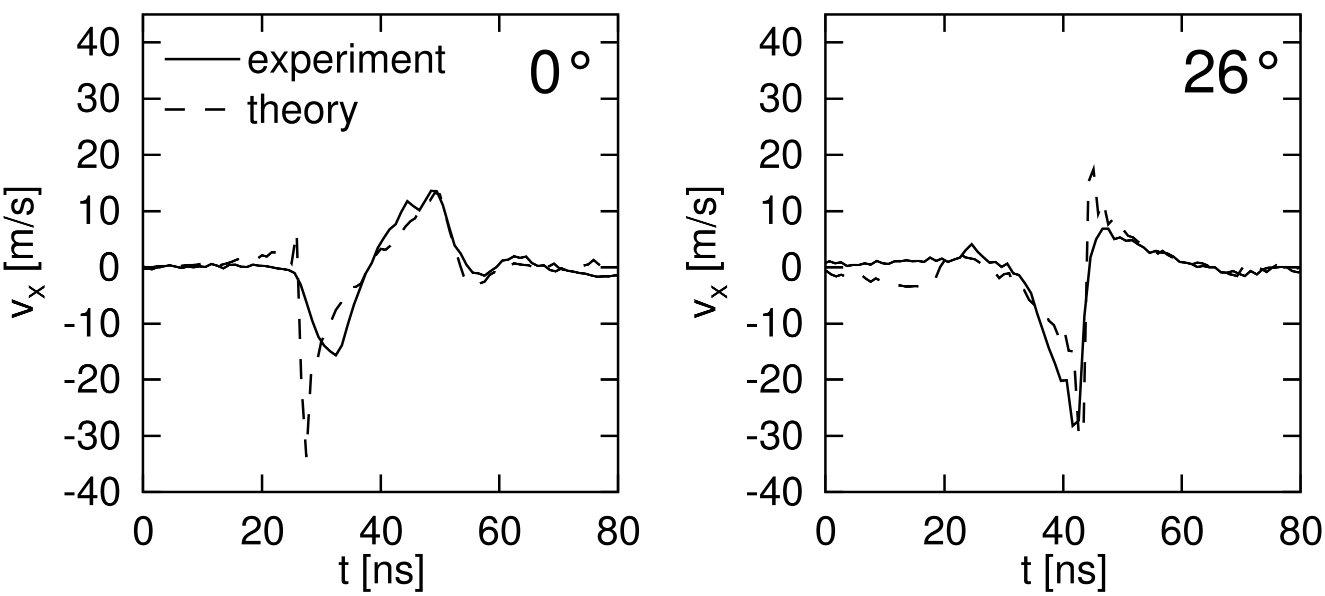
  Peak strain: 0.005 to 0.010 (near fracture)

EXPERIMENT FOR SI (001)

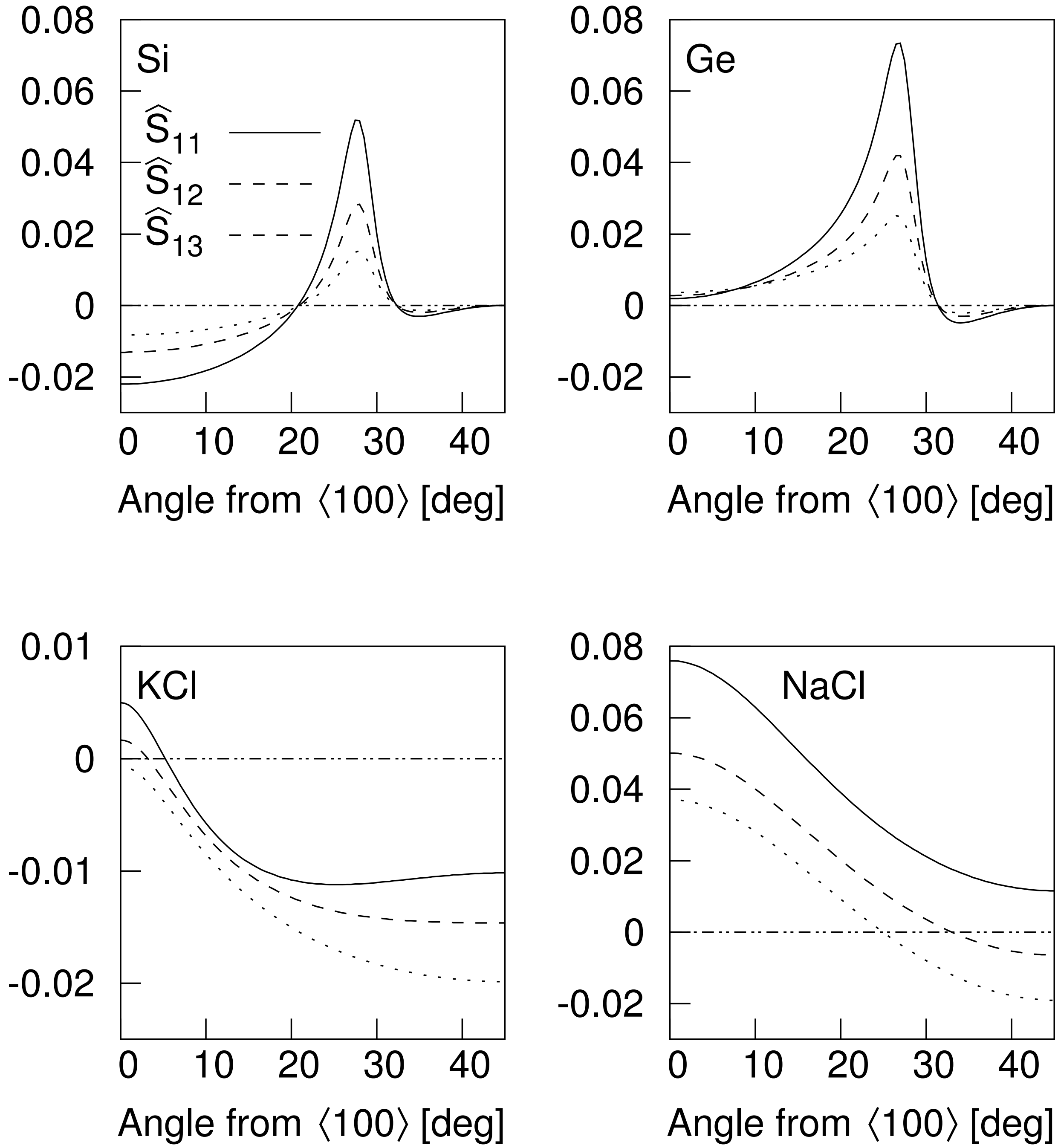
Longitudinal velocity waveforms at close location:



Longitudinal velocity waveforms at remote location:



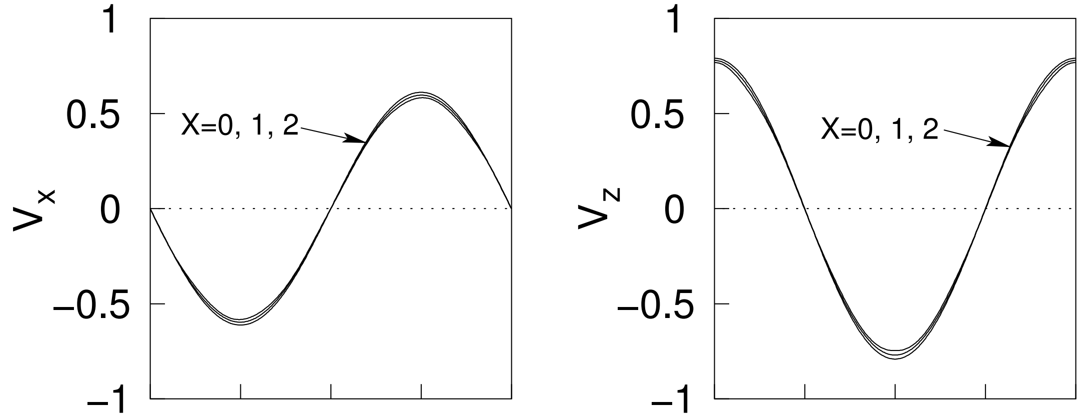
NONLINEARITY MATRICES IN (001) PLANE



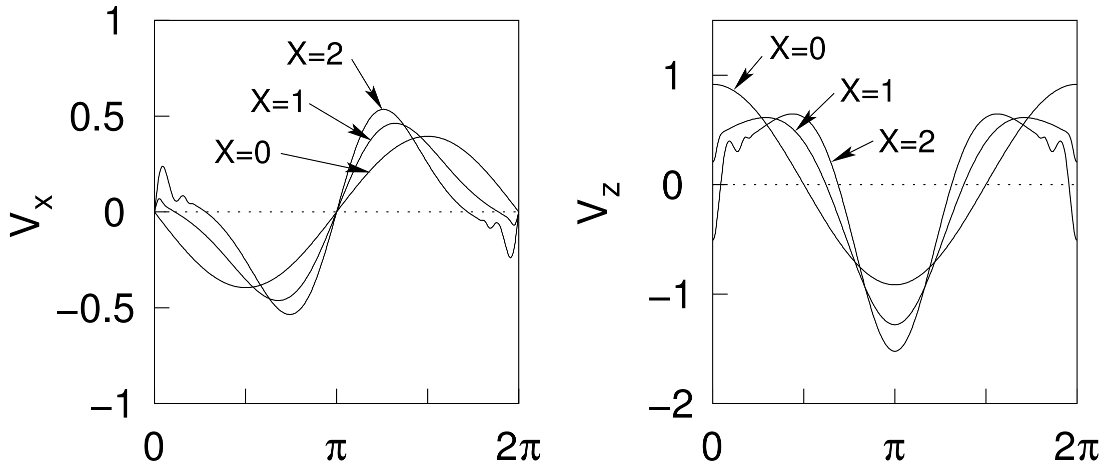
Other crystals investigated:
RbCl, BaF_2, CaF_2, SrF_2, Al, Ni, Cu, C (diamond)

OTHER EFFECTS IN (001) PLANE

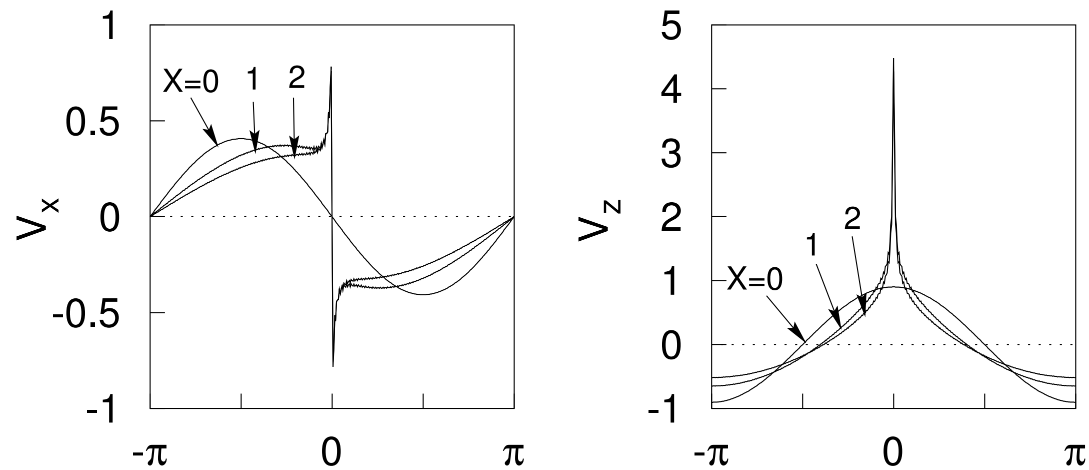
Si (001), $\theta \approx 21^\circ$: $\widehat{S}_{lm} \approx 0$



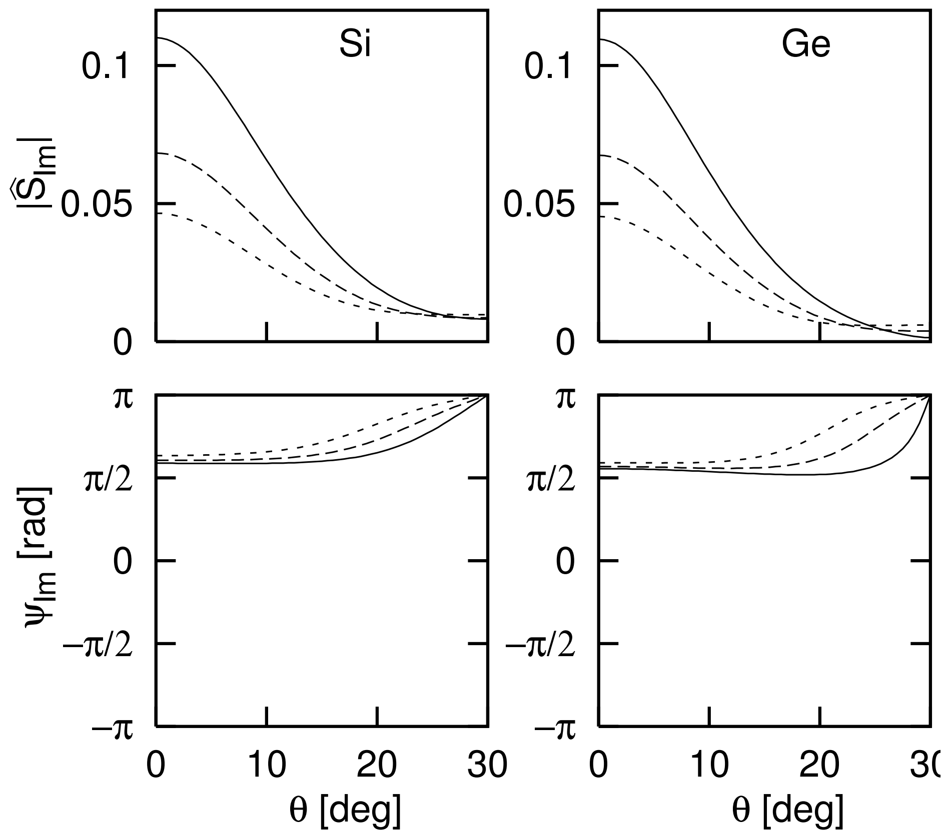
KCl (001), $\theta \approx 3^\circ$: $\widehat{S}_{11} > 0$, $\widehat{S}_{12} \approx 0$, $\widehat{S}_{13} < 0$

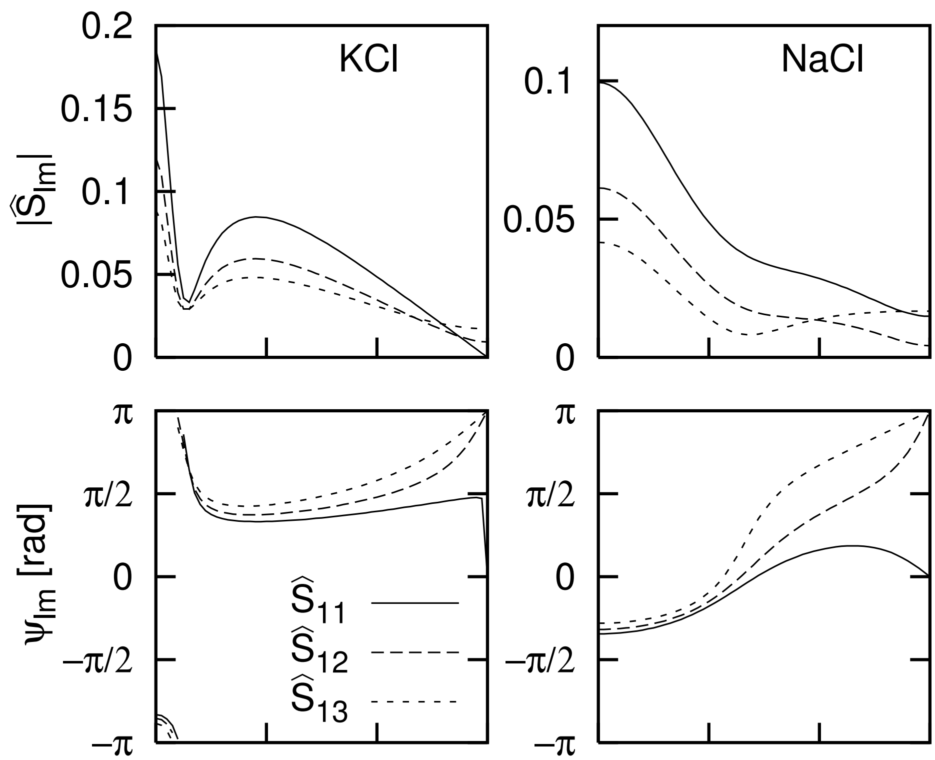


KCl (001), $\theta \approx 10^\circ$: $|\widehat{S}_{11}| < |\widehat{S}_{12}| < |\widehat{S}_{13}|$



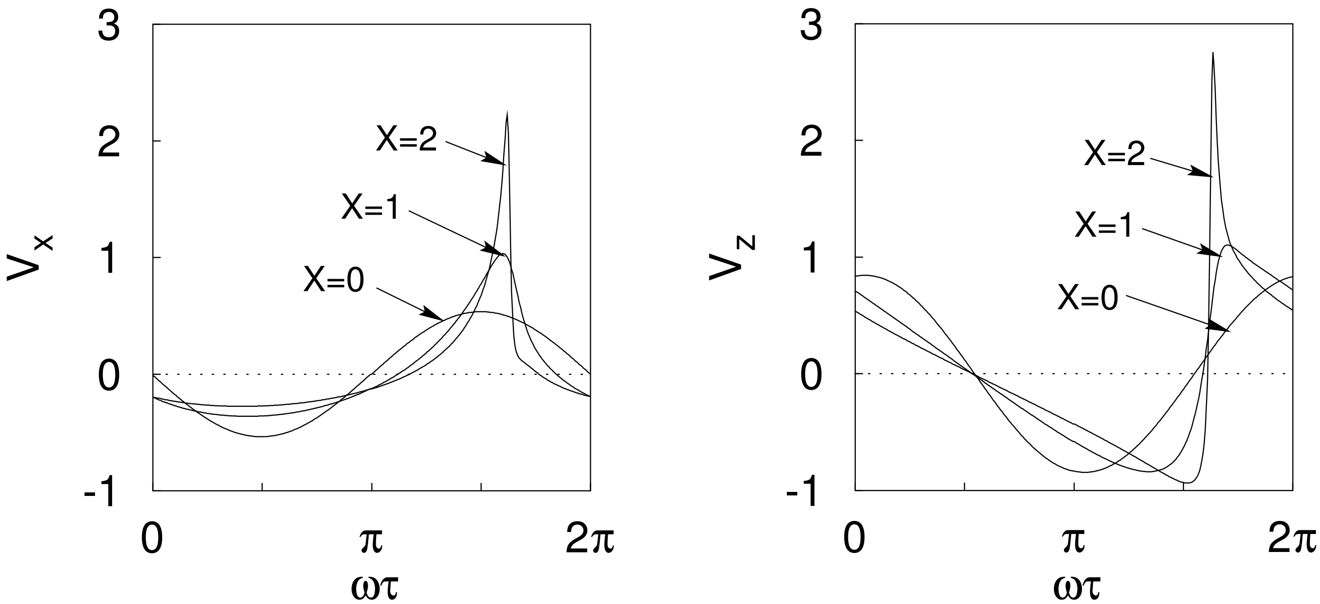
NONLINEARITY MATRICES IN (111) PLANE



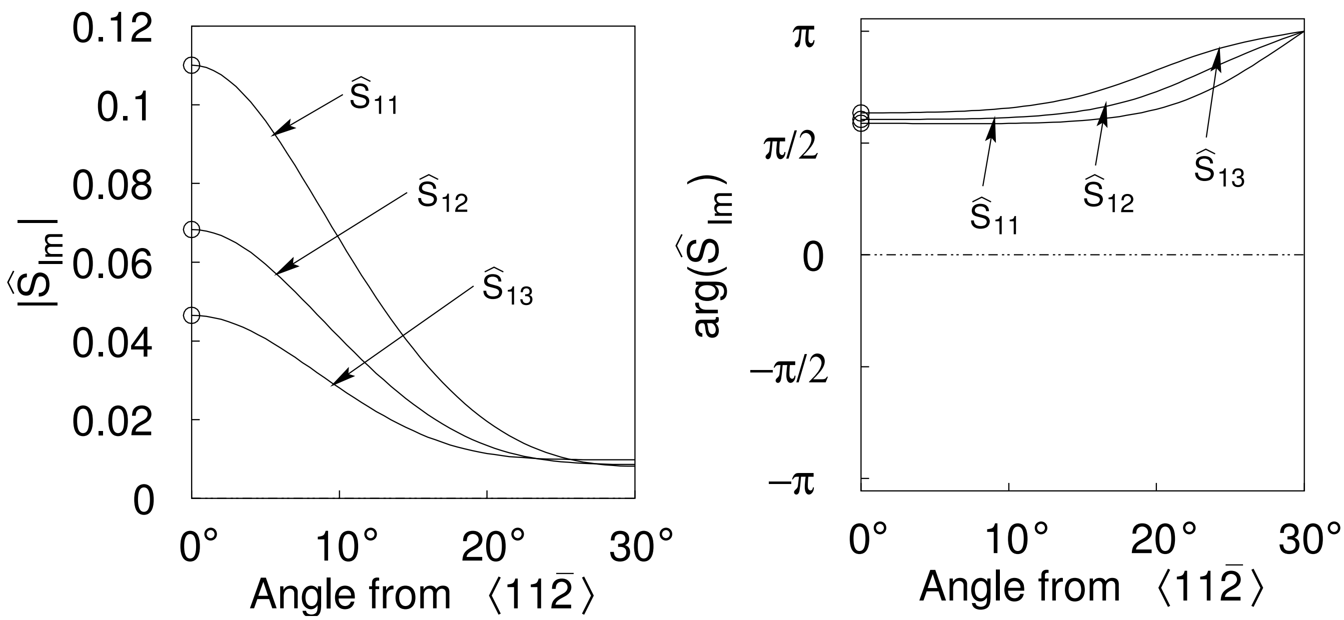


SIMULATIONS WITH SINUSOIDS FOR SI (111)

Velocity waveform in direction 0° from $\langle 11\bar{2} \rangle$:



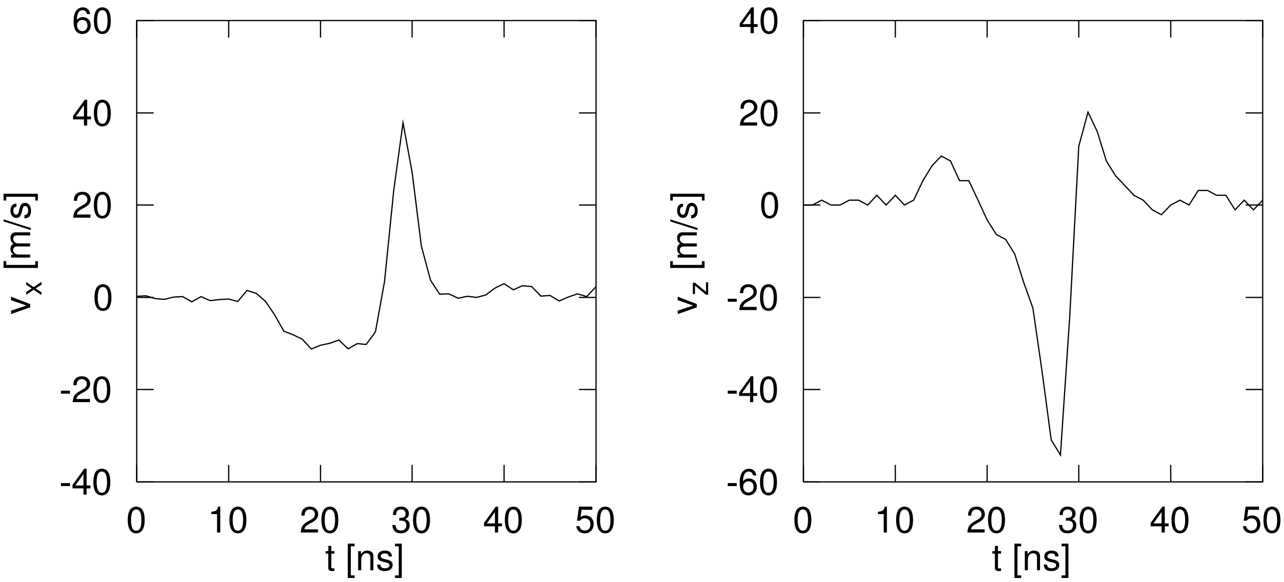
Nonlinearity matrix elements:



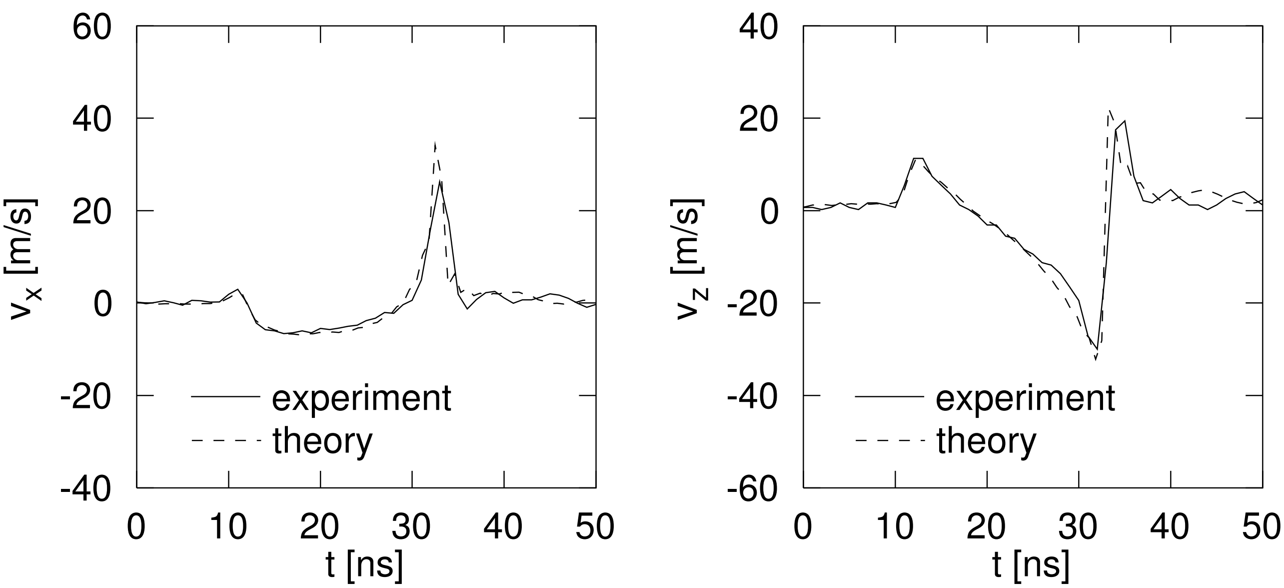
EXPERIMENT FOR SI (111)

Propagation in direction 0° from $\langle 11\bar{2} \rangle$

Velocity waveforms at close location:



Velocity waveforms at remote location:



PHASE OF MATRIX ELEMENTS

Define the matrix element transformation

$$\widehat{S}_{lm}^{\psi} = \widehat{S}_{lm} e^{i(\operatorname{sgn} n)\psi}\,, \qquad n = l + m\,.$$

The evolution equations with the transformed matrix are

$$\begin{aligned}
\frac{dv_n^{\psi}}{dx} &= -\frac{n^2 \omega_0 c_{44}}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \widehat{S}_{lm}^{\psi} v_l^{\psi} v_m^{\psi} \\
&= -\frac{n^2 \omega_0 c_{44}}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \widehat{S}_{lm} e^{i(\operatorname{sgn} n)\psi} v_l^{\psi} v_m^{\psi}
\end{aligned}$$

If the transformed spectral amplitudes are

$$v_n^{\psi} = v_n e^{-i(\operatorname{sgn} n)\psi}\,,$$

then it follows that

$$\frac{dv_n}{dx} = -\frac{n^2 \omega_0 c_{44}}{2\rho c^4} \sum_{l+m=n} \frac{lm}{|lm|} \widehat{S}_{lm} v_l v_m\,.$$

Result: Equations relate phase ψ of matrix elements to spectral amplitudes (and thus waveforms).

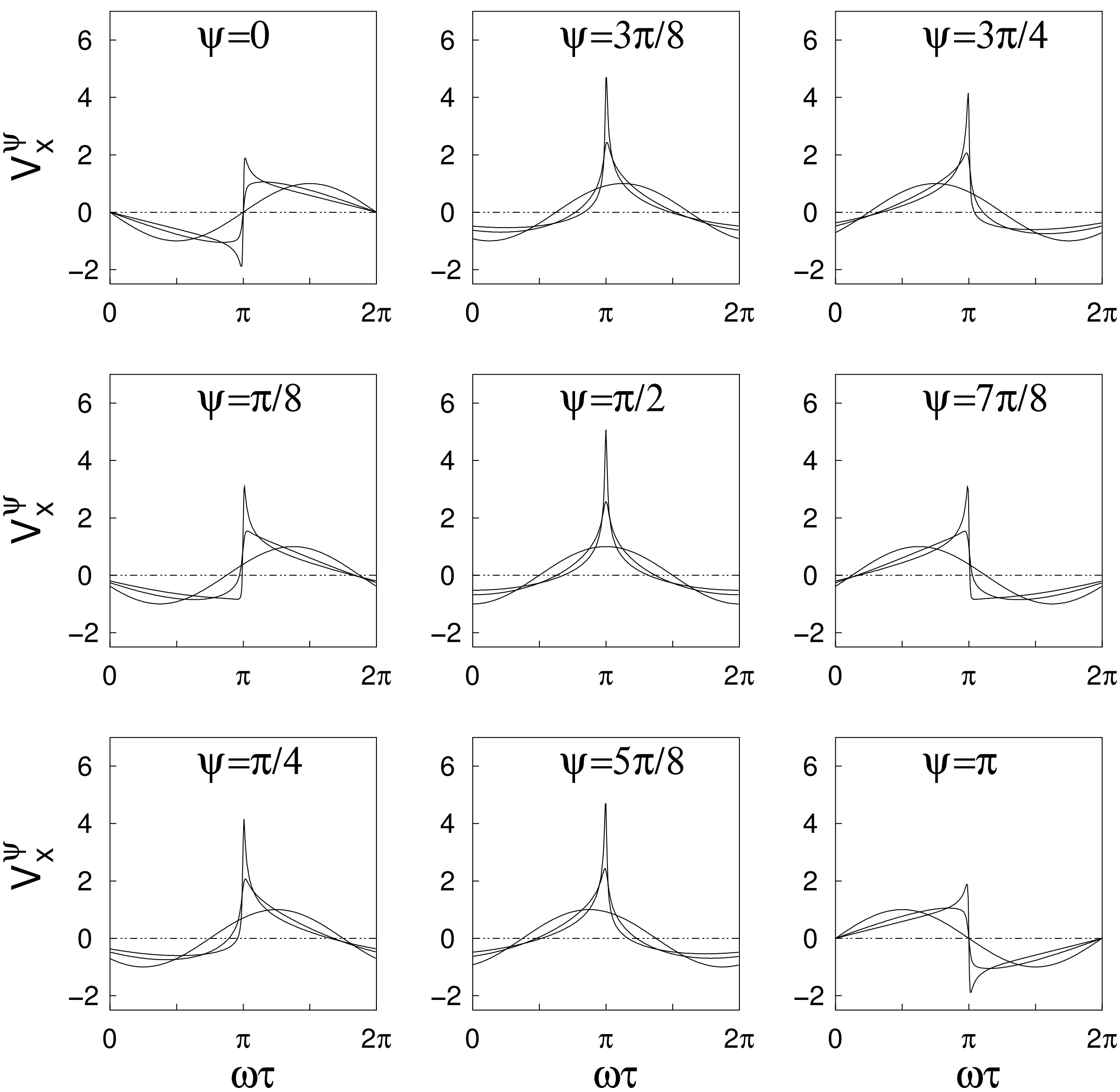
PHASE TRANSFORMED WAVEFORMS

Consider artificial example of Rayleigh waves in steel

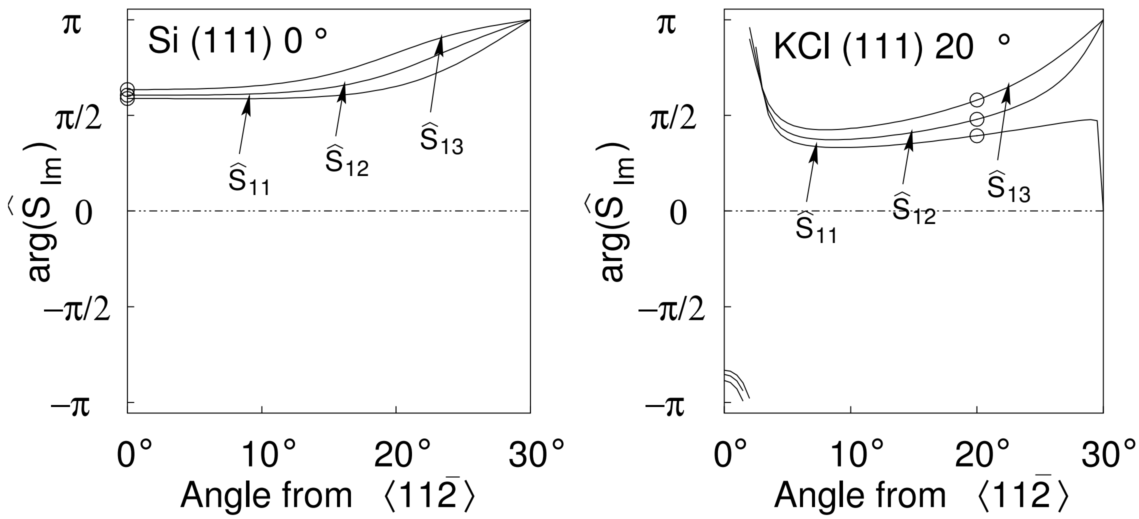
$$\widehat{S}_{lm} \quad \longrightarrow \quad \text{positive, real valued matrix elements}$$
$$v_n \quad \longrightarrow \quad \text{corresponding spectral components}$$

under the transformation $v_n^{\psi} = v_n e^{-i(\operatorname{sgn} n)\psi}$.

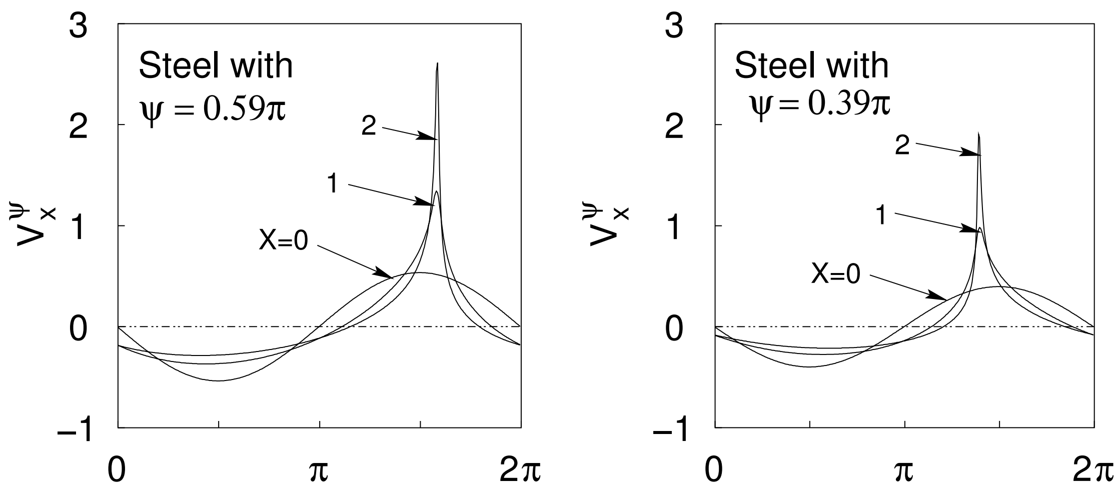
Resulting longitudinal velocity waveforms:



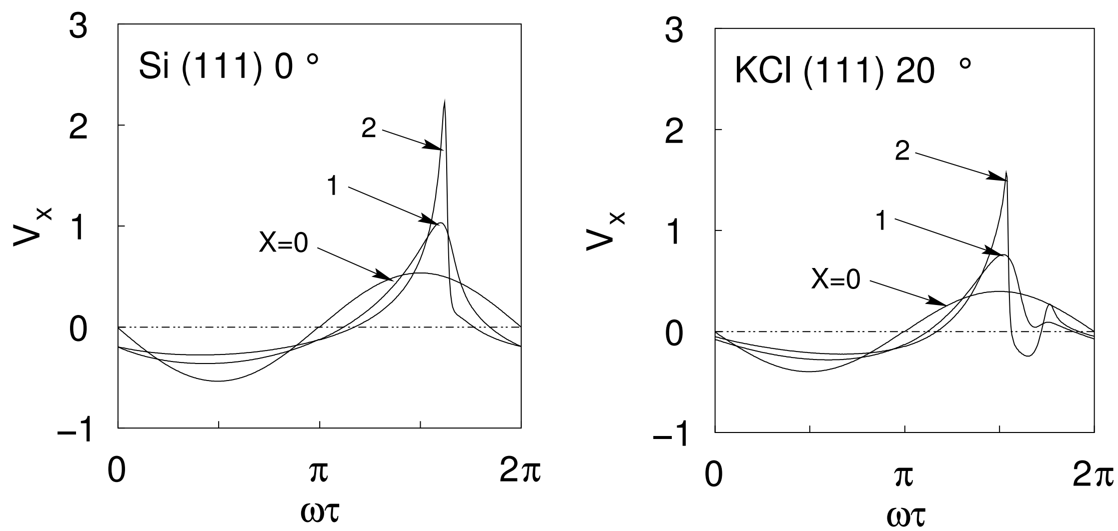
APPROXIMATE AND FULL SOLUTIONS



Approximation:



Full Solution:



SUMMARY

Results:

- Investigations of nonlinear SAWs in cubic crystals for:
  - Different materials, cuts, directions
- Nonlinearity matrix is useful for characterizing waveform distortion.
- In (001) plane:
  - Sensitive dependence of nonlinearity on direction
  - Compression and rarefaction shocks in same plane
  - Shock suppression in certain directions
  - Agreement between measured and calculated waveforms
- In (111) plane:
  - Asymmetric waveform distortion
  - Low frequency oscillations due to phase differences
  - Agreement between measured and calculated waveforms
  - Development of approximate method which works well where phases between elements are similar